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APPLICATION OF FUZZY RELATIONS TO TEST THEORY

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На відміну від класичного ймовірного підходу, в даній статті розглядається метод генерування та оцінки тестів, заснований на нечіткому підході. Це призводить до завдань, які можуть бути вирішені в рамках нечітких реляційних рівнянь. Кілька прикладів ілюструють користь такого підходу.
Ключові слова: теорія тестів, генерування та оцінка тестів, нечіткі реляційні рівняння.

В отличие от классического вероятного подхода, в данной статье рассматривается метод генерирования и оценки тестов, основанный на нечетком подходе. Это приводит к задачам, которые могут быть решены в рамках нечетких реляционных уравнений. Несколько примеров иллюстрируют пользу такого подхода.
Ключевые слова: теория тестов, генерирование и оценка тестов, нечеткие реляционные уравнения.

Unlike the classical probability-based approach we consider the ge-neration and evaluation of tests based on a fuzzy approach. This leads to tasks which can be solved within the frame of fuzzy relational equations. Several examples illustrate the usefulness of our approach.
Keywords: test theory, generation and evaluation of tests, fuzzy relational equations.

Generation of Tests with Desired Properties

Tests are one of the powerful means in modern educational systems [2]. The structure of a test is determined by items which are characterized by complexity, discrimination, correlation to the test and so on. Items are usually collected into so-called item banks that can be used for the generation of different tests. The test has to be designed from items that have desired characteristics according to test specification. The test examines the knowledge of a testee with respect to some subject, the latter being characterized by units of knowledge (UOK). Obviously, each item can be interrelated with a set of UOK.

One of the problems of test developers is the generation of a test from the item bank that has certain statistical characteristics (according to test specification) as well as a desired unit of knowledge (according to the subject that is assessed). There may be situations when it is necessary to design the tests from one subject but for different groups with different levels of knowledge (Fig.1).

The problem of choosing items is complex, because the bank of items may contain up to some thousands objects that are collected at universities or national centers of assessment. The scheme of test generation is shown in Fig.2.

Formalization of the Task and Problem Formulation

Let us consider an item bank containing N items $T = \{I_1, I_2, \ldots, I_N\}$ from some subject (e.g., mathematics). Moreover, we have M UOK $U = \{U_1, U_2, \ldots, U_M\}$ describing this subject (e.g., numbers, sets, functions, statistics, geometry,…). Let R express the quantification of the relation between the items and the UOK reflecting the fitness of the items with respect to these units:
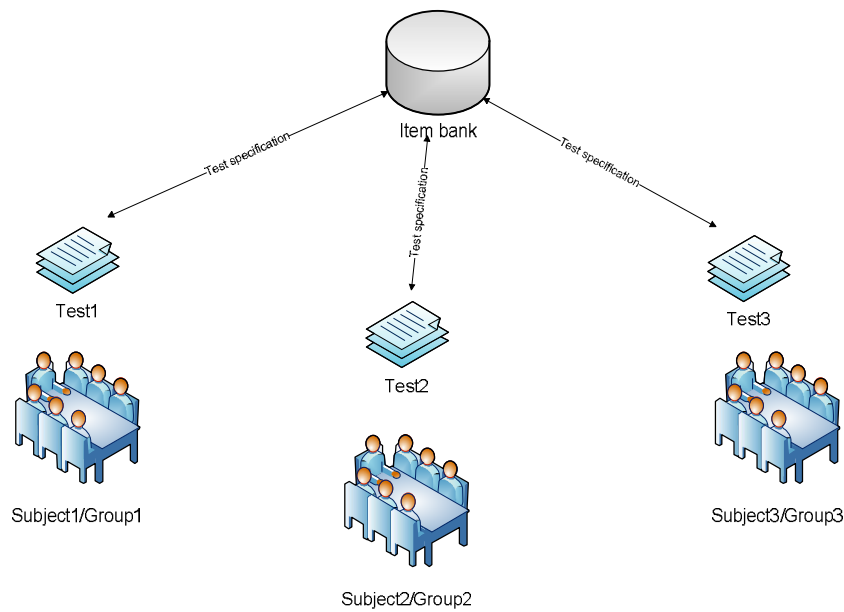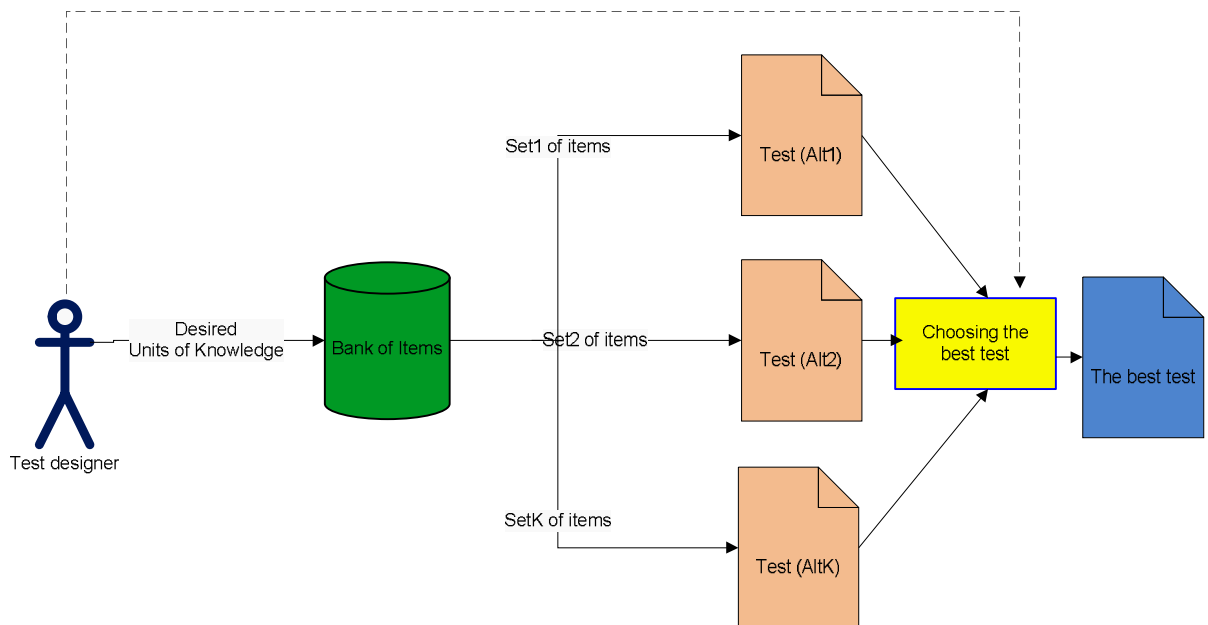
Fig. 1. Working with item bank

Fig. 2. Procedure of test generation

$$R = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1M} \\ r_{21} & r_{22} & \dots & r_{2M} \\ \dots & \dots & \dots & \dots \\ r_{N1} & r_{N2} & \dots & r_{NM} \end{pmatrix}.$$

The elements r_{ij} may be from the unit interval (i.e., R can be interpreted as fuzzy relation) expressing the truth degree of the fitness. Sometimes, however, it is useful to have the r_{ij} from a lattice, e.g. from set $\{0,1,\dots,S\}$. In this case the matrix elements estimate the level of correspondence of fitness. In what follows, however, we assume the unit $[0,1]$ as basis for evaluation.

There are at least two problems to consider. First, one has to find the underlying set of UOK U^* when the testee has performed his test T^* and got the results as truth levels of answers with respect to the items. Hence, we answer the question which UOK does the testee know well. This is the *direct* problem. Second, one may be faced with the question how to choose the set of items T^* from the item bank (i.e., the test) if we want to test some subset U^* of UOK. It is clear that we may get different tests which assess the same set of UOK. This is called the *inverse* problem.

The sets T^* and U^* are supposed to be fuzzy sets on their universes T and U. The memberships are denoted by small letters and for simplicity we

equate the fuzzy sets with their membership vectors, i.e. $T^* = \left(i_1^*,...,i_N^*\right)$, $U^* = \left(u_1^*,...,u_N^*\right)$.

The direct problem solution

Let $T^* = \left\{i_1^*, i_2^*,...,i_N^*\right\}$ is the result of the test for some testee. Using relation R and T^* we can find the appropriate fuzzy set for the UOK successfully handled by the testee by computing

$$U^* = T^* \circ R \qquad (1)$$

where "$\circ$" means the max-min composition law for fuzzy relations and sets, i.e.

$$u_j^* = \max_{k \in \{1,...,N\}} \min\left(i_k^*, r_{kj}\right), j \in \{1,...,M\}. \quad (2)$$

Example 1. Let us consider a test in mathematics containing of 10 items assessing the following units of knowledge: u_1 - Algebra, u_2 - Numbers and Expressions, u_3 - Equations and Inequalities, u_4 - Functions, u_5 - Combinatorial Calculus and Probabilities, u_6 - Statistics, u_7 - Geometry, u_8 - Plane Geometry, u_9 - Stereometry.

Moreover, we have the relation R (obtained from experts) between items and units of knowledge

$$R = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

Assume that the testee has obtained the following result: $\left(i_1^*, i_2^*,...,i_{10}^*\right) = (1,0,1,1,0,0,0,1,0,0)$. Then computation (2) yields $U^* = (1,1,1,1,0,1,1,1,0)$. It means that the testee knows u_1 - u_4 and u_6 - u_8, but he does not know u_5 and u_9.

Now let the answers be evaluated from a 5-degrees scale, e.g. from the set $\{0, 0.25, 0.5, 0.75, 1\}$ and suppose that the testee got the following result: $\left(i_1^*,...,i_{10}^*\right) = = \left(0.75, 0, 0.75, 1, 0.25, 0, 0.25, 0.5, 0.25, 0.25\right)$. According to (2) we find $U^* = \left(0.75, 0.75, 0.75, 1, 0, 0.75, 1, 1, 0.25\right)$. This means that the testee does not know only u_5 and knows the remaining units at different levels.

The Inverse Problem

The inverse problem consists in the determination of T^* with known R, U^* in (1). That is we want to know which tests might have led to the evaluation U^*. This task is much harder to solve (in comparison to the direct problem) and we

may be faced with infinitely many solutions or no solution at all. It is a classical problem in the theory of fuzzy relation equations [3,5,7]. In the case of solveability the maximal (in the sense of fuzzy inclusion) solution $\hat{T} = \left(\hat{t}_1,...,\hat{t}_N\right)$ is given by

$$\hat{T} = R\,\alpha\,U^*. \qquad (3)$$

Where $\left(R\,\alpha\,U^*\right)_k = \min_{1 \le j \le M} r_{kj}\,\alpha\,u_j^*$, and the well-known α-operation (Goedel implication) is defined as

$$a\,\alpha\,b = \begin{cases} 1 & \text{for } a \le b, \\ b & \text{otherwise}. \end{cases}$$

There may be, however, a large number of minimal solutions [7] the calculation of which is not trivial for larger N (typical in test theory).

Example 2. Let us consider the test with 10 items and relation from Example 1. Now we want to find the assessment of answers to items if we are given the UOK by $U^* = (0.75, 0.75, 0.75, 1, 0, 0.75, 1, 1, 0.25)$. We obtain the maximal solution

$\hat{T} = (0.75, 0, 0.75, 1, 0.25, 0, 0.25, 0.75, 0.75, 0.75)$ and the four minimal solutions

$$T_1^{\min} = (0.75, 0, 0.75, 1, 0, 0, 0, 0, 0, 0),$$
$$T_2^{\min} = (0.75, 0, 0, 1, 0, 0, 0, 0.75, 0, 0),$$
$$T_3^{\min} = (0, 0, 0.75, 1, 0, 0, 0, 0, 0.75, 0),$$
$$T_4^{\min} = (0, 0, 0, 1, 0, 0, 0, 0.75, 0.75, 0).$$

Inverse Problems with Restrictions

Often the tester is not interested in the whole solution set of (1), but solutions with special properties are desired, as mentioned in Section 1. We distinguish two approa-ches: individual and global.

Individual Approach

In this case, we search at least one solution of (1) with T individual restrictions on the member values in each element I_j leading to the following task: Search T^* fulfilling

$$U^* = T^* \circ R$$
$$\underline{T} \subseteq T^* \subseteq \overline{T}, \qquad (4)$$

where $\underline{T}, \overline{T}$ are fuzzy sets on T and "$\subseteq$" means the inclusion of fuzzy sets.

This situ-ation occurs for example if we want to get a solution T^* where certain items are surpressed and other items are to be in the solution set with high evaluation.

In practice one is often faced with the problem to search for solutions with a special structure. Suppose, one has to determine a test $T = \{I_1, I_2,...,I_N\}$ where item I_j takes part with probability p_j, i.e. T is characterized by a probability distribution P. This restriction can be transformed into a fuzzy set T_P^* using corresponding methods [4,6]. Due to a certain

ambiguity in the choice of the transformation method and accounting that the p_j may be imprecise it seems to be more appropriate to include T_P^* in bounds, i.e. $\underline{T}_P \subseteq T_P^* \subseteq \overline{T}_P$ and we are led to task (4). The following statement enables the determination of a solution of (4) in an efficient way.

Statement 1. *Denote the solution set of (4) by* Γ. *Moreover let* $\tilde{T} = \hat{T} \cap \overline{T}$ *with* $\hat{T} = R \alpha U^*$ *(see (3)). Then* $\Gamma \neq \varnothing$ *iff* $\tilde{T} \in \Gamma$.

The proof follows from [9] where a more general situation is considered.

Example 3. Let U^* given as in Example 2. Suppose, we are interested in item soluti-ons with evaluations of at least 0.5 for items I_1, I_4, I_8, I_9. Items I_2, I_7, I_{10} are irrelevant and items I_3, I_5, I_6 should be excluded from consideration. This leads the restrictions
$\underline{T} = (0.5, 0, 0, 0.5, 0, 0, 0, 0.5, 0.5, 0)$,
$\overline{T} = (1, 1, 0, 1, 0, 0, 1, 1, 1, 1)$. One sees that $\tilde{T} = (0.75, 0, 0, 1, 0, 0, 0.25, 0.75, 0.75, 0.75)$ fulfills the restrictions and it is a solution, because $T_2^{\min} \subseteq \tilde{T} \subseteq \hat{T}$.

Global Approach
It may be of interest to globally confine the memberships of T^* to a given (crisp) sub-set $\Omega \subseteq [0,1]$. This situation is typical when Boolean solutions are desired $(\Omega = \{0,1\})$ or solutions where the membership of each item i should be below a level or above another one $(\Omega = [0, \underline{\omega}] \cup [\overline{\omega}, 1]$ with $0 \le \underline{\omega} \le \overline{\omega} \le 1)$. Formally this means that we search a T^* with
$$U^* = T^* \circ R, \qquad (5)$$
$$i_j^* \in \Omega \text{ for } j = 1, ..., N.$$

For the analysis of (5) we apply results given in [1]. Therefore define a function $\varphi_\Omega : [0,1] \to [0,1]$ by
$$\varphi_\Omega(a) = \sup_{\substack{b \in \Omega \\ b \le a}} b. \qquad (6)$$

Remark 1.
a) For $\Omega = \{0,1\}$ (Boolean case) we obtain
$$\varphi_\Omega(a) = \begin{cases} 1 \text{ for } a = 1, \\ 0 \text{ otherwise.} \end{cases}$$
b) For $\Omega = [0, \underline{\omega}] \cup [\overline{\omega}, 1]$ as above we have
$$\varphi_\Omega(a) = \begin{cases} a \text{ for } a \in \Omega, \\ \underline{\omega} \text{ otherwise.} \end{cases}$$
A solution of (5) can be found by the following

Statement 2. *Denote the solution set of (5) by* Ψ_Ω *and let* Ω *be closed. Set* $\hat{T} = \varphi_\Omega(\hat{T})$ *(i.e.* φ_Ω *applied elementwise). Then* $\Psi_\Omega \neq \varnothing$ *iff* $\hat{T} \in \Psi_\Omega$.

Example 4. Suppose U^* to be like in Example 2. We want to determine a solution with evaluations not lower than 0.5. Otherwise we exclude the item from further consideration. That is, $\Omega = \{0\} \cup [0.5, 1]$. A solution fulfilling the constraints is $\hat{T} = (0.75, 0, 0.75, 1, 0, 0, 0, 0.75, 0.75, 0.75)$, and obviously $T_1^{\min} \subseteq \hat{T} \subseteq \hat{T}$.

Conclusion

The proposed approach of analysis and the formation of tests based on fuzzy relations opens up prospects for the automation of test generation based on the matrix elements of knowledge regarding the relationship and bank of items. Taking into account that in real test systems the item bank may contain hundreds of items, the problem of determining an optimal set of items is important. However, the demand for exact solvability may be too restrictive (i.e. Γ or Ω may be empty). Then one might search for approximative solutions (e.g. by transforming U^* into an interval-valued fuzzy set, see [8]). This will be the topic of future research.

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MODELING A BOILING PROCESS UNDER UNCERTAINTIES

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Теплообмін у процесі кипінні залежить від цілої низки чинників (стадії кипіння, фізичних та геометричних параметрів, змінних стану і умови обтікання). Всі варіанти засновані на емпіричних співвідношеннях, отриманих на базі експериментальних даних, оскільки досі відсутня узагальнена теорія темплообміну. Із збільшенням перегріву стіни (Tw-Ts) випаровування змінюється від конвективного кипіння до ядерного, а відтак – до film-кипіння. Для розрахунку коефіцієнта тепловіддачі кожного типу кипіння існують окремі рівняння. Існує можливості враховувати невизначеність в динамічному моделюванні кипіння. Для цього була розроблена нечітка модель типу Такагі-Сугено, яка містить нечіткі переходи між стадіями кипіння.
Ключові слова: процес кипіння, теплообмін, динамічне моделювання кипіння, нечітка модель Такагі-Сугено.

Теплообмен при кипении зависит от целого ряда факторов (стадии кипения, физических и геометрических параметров, переменных состояния и условия обтекания). Все варианты основаны на эмперических экспериментальных данных, так как до сих пор отсутствует общая теория теплообмена. С увеличением перегрева стены (Tw-Ts) испарение изменяется от конвективного кипения до ядерного, а затем до film-кипения. Для расчета коэффициента теплоотдачи каждого типа кипения существуют отдельные уравнения. Существует возможность учитывать существующую неопределенность в динамическом моделировании кипения. В связи с этим была разработана нечеткая модель типа Такаги-Сугено, включающая в себя нечеткие переходы между стадиями кипения.
Ключевые слова: процесс кипения, теплообмен, динамическое моделирования кипения, нечеткая модель Такаги-Сугено.

The heat transfer during boiling depends on a variety of factors (boiling stage, material parameters, geometrical parameters, state variables and flow conditions). All variations are based on empirical relationships gained from experimental data because there is still no comprehensive theory. With increasing wall superheat (Tw-Ts), the evaporation changes from convective boiling to nucleate boiling and then to film boiling. For each type of boiling, separate equations for the calculation of the heat transfer coefficient do exist. This paper presents possibilities to take account of the existing uncertainties in the dynamic simulation of boiling. For this reason a Takagi-Sugeno-fuzzy-model was developed which includes the fuzzy transitions between the boiling stages.
Key words: the process of boiling, heat transfer, dynamic simulation of boiling, fuzzy model of Takagi Sugeno.

Introduction. Steam production is the basic process for the generation of electrical energy in nuclear or coal power plants. By means of the flow type, the boiling process can be distinguished between pool boiling and flow boiling. Pool boiling occurs in free flow. In a forced flow the evaporation process is called flow boiling. Undercooled boiling is not investigated. The medium has reached its saturation temperature.

The boiling process can be subdivided in three stable stages:

– convective boiling,

– nucleate boiling

– and critical boiling states (film boiling or dryout of the heated surfaces).

The transitions between the boiling stages are not sharp and they are determined by the wall superheat ΔT. The wall superheat is the difference between the wall temperature T_W and the saturation temperature of the fluid T_S. The saturation temperature remains constant in the boiling process. The heat transfer is determined by the heat transfer coefficient α. The correlations are shown in the following figure. The abscissa is the logarithmic temperature difference (T_W-T_S) and the ordinate shows the heat flux density.

The boiling process begins with convective boiling. The wall temperature is only a few Kelvin above the saturation temperature. No or only very few bubbles are formed. The increase of the wall temperature increases the bubble formation and the convective boiling changes over to nucleate boiling. Point A marks the onset of nucleate boiling, ONB [1].

The bubbles form on cavities or scratches on the surface containing pre-existing gas/vapor nuclei. The rising bubbles mix the fluid and thus improve the heat transfer. This is demonstrated by the growing increase of the heat transfer coefficient. Further increase of the heat supply causes also an increase in bubble formation and the flow of fluid to the wall is lower. At a maximum heat flux density (point C, called the critical heat flux CHF) forms a closed vapor film restraining the heat transfer. This means that the transferred heat flow decreases, then reaches a minimum (Leidenfrost point) and increases. This behavior is achieved by setting the wall temperature. The evaporation process in nuclear or coal power plants is setting the heat flow. This means that due to the suddenly worsened heat transfer, the wall
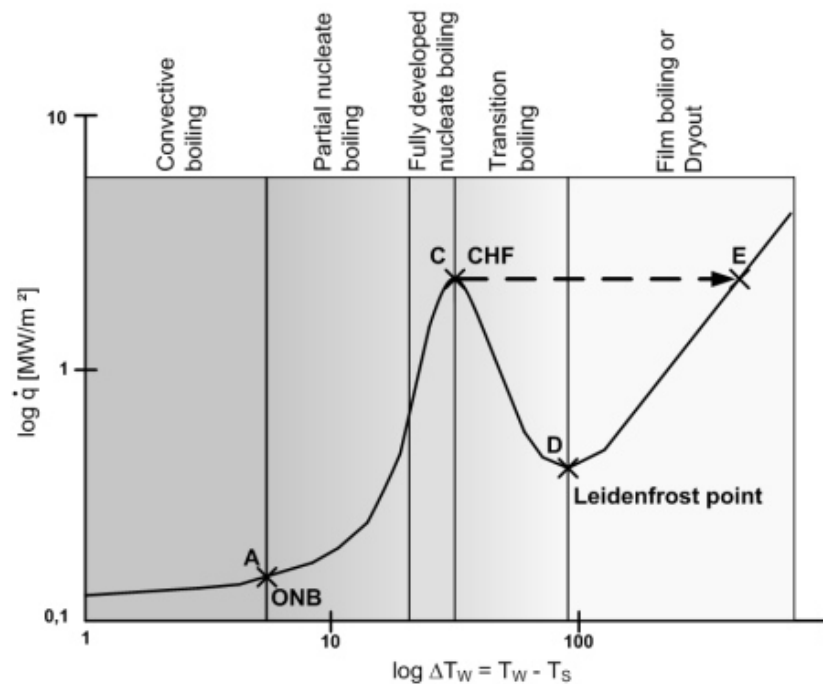
Figure 1 – Explanation of the boiling stages based on the boiling curve of Nukiyama

temperature increases drastically (dashed line from C to E) [2].

Mathematical description of the boiling process. The describing variable for the modeling of the heat transfer is the heat transfer coefficient α. For each boiling stage many formulas do exist which mostly are based on experimental data. Besides viscosity, thermal conductivity, density, thermal expansion coefficient and geometrical parameters the heat transfer depends on many influencing factors. During heat transfer with phase change, the number of variables is extended by enthalpy of vaporization, saturation temperature, vapor density and surface tension. Microstructure and material of the heating surface are relevant as well. The multitude of influencing factors and their complex interaction are the cause that no comprehensive theory could be developed yet. Thus, all mathematical calculations are based on empirical or semi-empirical relations [2].

Convective boiling
The calculation of the heat transfer coefficient α is based on the equations for forced convective heat transfer [2].

Nucleate boiling
The heat transfer coefficient for nucleate boiling is described by empirical models. The equation refers to a standard state with α_0 and $\dot{q}_0$ and considers the relative effects of wall roughness C_W, the boiling pressure by $F(p^*)$ and n and the pipe diameter [2].

Critical boiling states
Film boiling
The heat transfer coefficient for film boiling is composed of the heat transfer coefficient α_L and

α_S. The heat transfer coefficient α_L is determined by the heat conduction process through the vapor layer and α_S through the heat radiation process [2]. The explicit equation of Bromley's proximity for the technical area of interest is defined in [4].
Dryout
The calculation of the heat transfer coefficient α is based on the equations for forced convective heat transfer [2].

Takagi-Sugeno-Fuzzy-Model. To model the fuzzy transition between the boiling stages, a Takagi-Sugeno fuzzy model is suitable. The height of the wall superheat is the decision criterion, which boiling stage is present. For this reason it is fuzzified and ΔT is defined as a linguistic variable (Figure 2), which consists of three membership functions "small", "medium" and "large". Convective boiling is definitely given if the wall superheat is located between 0 K and 7 K ("small"), we speak of nucleate boiling , when ΔT is between 20 K and 35 K ("medium") and critical boiling states starts at a high temperature of 100K ("large"). The transition regions are modelled linearly. The data were taken from Figure 1. The decision which mechanism works of the critical boiling states is based on the critical vapor content. If the vapor content x is low, then it is film boiling, is it "high" then works the heat transfer mechanism dryout of the heating surface.

The following rule base is derived:
If ΔT = "small" then α = convective boiling
If ΔT = "medium" then α = nucleate boiling
If ΔT = "large" and x = "low"
    then α = film boiling
If ΔT = "large" and x = "high"
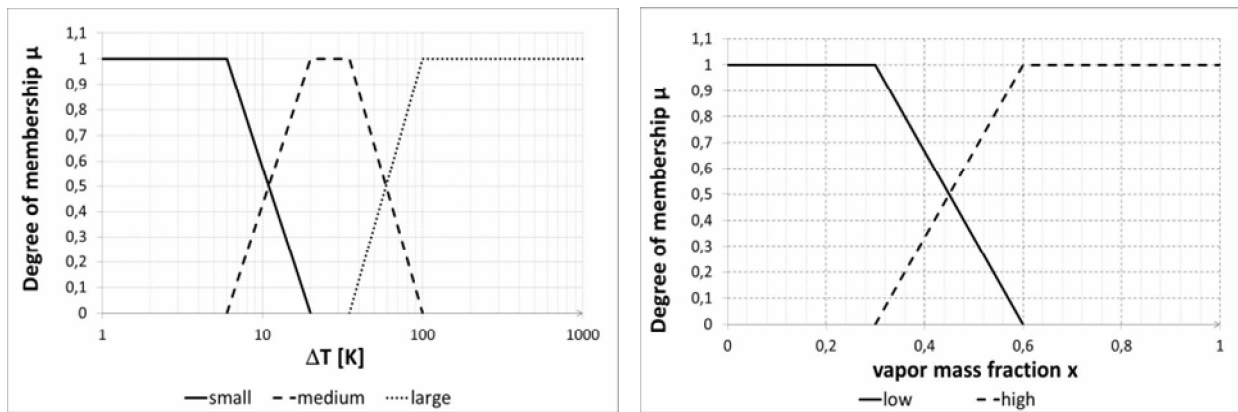   then α = film boiling   (1)

Figure 2 – Linguistic variable "wall superheat ΔT" and "vapor mass fraction x"

Subsequently, the accumulation is performed to determine the resulting heat-transfer coefficient.

Modeling a boiling process. To illustrate the methods / procedures developed, the following boiling process is applied (Figure 3). In an electrically heated thin-walled pipe is saturated water supplied and vaporized over the pipe length. The dynamic behavior is described by following simplified non-linear differential equation system (Assumption: the pressure is constant).
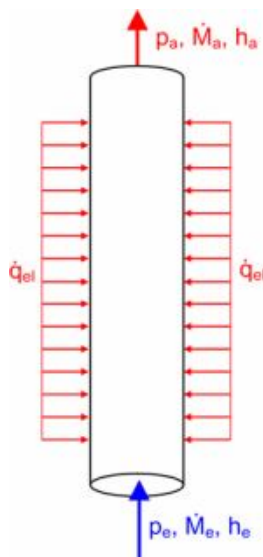
p_e, p_a – Inlet- outlet pressure;
$\dot{M}_e, \dot{M}_a$ – Inlet- outlet mass flow;
h_e, h_a – Inlet- outlet enthalpy;
m_W – Pipe mass;
c_W – Heat capacity of pipe wall;
m – Mass of the medium;
T_W – Wall temperature;
$\dot{q}_{el}$ – Electrical heat flux

Figure 3 – Model for describing a boiling process

Heat balance medium:

$$\left(q_m \cdot V + V \cdot h_e \cdot \frac{\partial q}{\partial h} - V \cdot h_m \cdot \frac{\partial q}{\partial h} \right) \frac{dh_m}{dt} + 2\dot{M}_e \cdot h_m =$$
$$= 2\dot{M}_e \cdot h_m + \alpha \cdot A \cdot \left(T_W - T_{M_m} \right). \quad (2)$$

Heat balance pipe:

$$m_w \cdot c_W \cdot \frac{dT_W}{dt} = Q_{ei} - \alpha \cdot A \cdot \left(T_W - T_{M_m} \right). \quad (3)$$

The realization of the model equations is carried out with a computer algebra system. The heat flux density is given. After 10 seconds the heat flux density increases (Figure 4, left). At the beginning of the simulation, the fluid temperature is already at saturation temperature. This value remains constant throughout the boiling process. The simulation starts with the state of convective boiling. The heat transfer coefficient is low. For the first 10 seconds a steady state appears. The wall temperature is a few Kelvin above the saturation temperature.

With increasing heat flux density the wall temperature rises. Due to the low heat transfer coefficient, the wall temperature initially rises steeply. With the onset of bubble production the water is getting mixed and the heat transfer improves. This means that the heat transfer coefficient increases and the wall temperature rises slower despite constantly increasing heat supply. The heat transfer to the water improves until the critical point is reached, so that the bubbles form a closed vapor film. The steam has considerably poorer heat transfer properties. That means for a given heat flux density, that the wall temperature (Figure 5) changes almost in a jump-like way and the process switches to stable film boiling. In right Figure 4 shows the heat flux over the wall superheat. When the critical heat flux is reached, then the wall superheat rises steeply.

Summary and Outlook

The article presents a possibility to take account of an aspect of model uncertainty. In a first step, the heat transfer coefficient for the dynamic simulation of boiling process is realized with a Takagi-Sugeno-fuzzy-model and integrated into the differential equation system. The dynamic simulation of the process example represents the process behavior realistically.

To extend the model, the parameter uncertainties for the major variables (e.g. pressure, temperature) are considered as fuzzy and in the simulation.
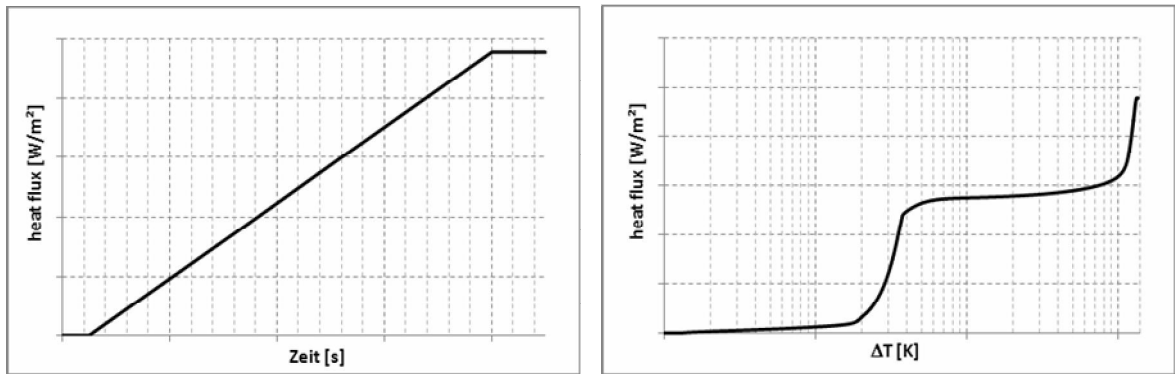
Figure 5 – Temporal course of the prescribed heat flux (left) and qualitative trend of the heat flux over the wall superheat T (right)
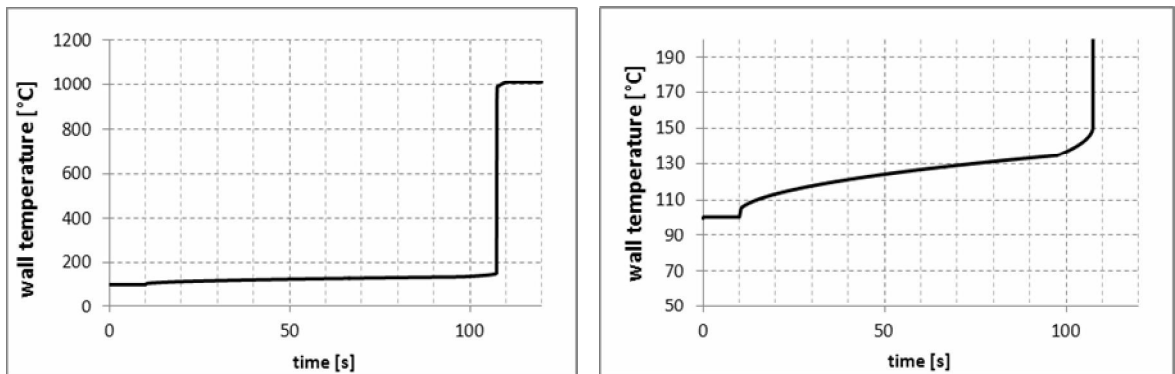

Figure 5 – Qualitative temporal course of the wall temperature, left) overview right) zoomed

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