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OPTIMAL CONTROL IN THE SWITCHED FUZZY MODELS OF MANAGEMENT PROCESSES

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Розглядаються проблеми проектування мінімальна енергія управління для одного класу динамічних систем Такагі-Сугено у безперервному часі. Робота спрямована на виконання комутаційного закону для підмоделі. Пропонований підхід заснований на принципі оптимальності Беллмана. Порівняння чітких перемикань і нечітких моделей проводиться з метою демонстрації ефективності нечіткої моделі для оптимізації індексу енергетичної ефективності. Ця гра з чіткими і нечіткими частинами для оптимізації проблеми обговорюється як майбутній актуальний напрямок досліджень. Всі результати проілюстровані на прикладах нечітких систем обчислювального інтелекту.
Ключові слова: управління перемиканням, Такагі-Сугено моделі, нечітка динамічна система, індекс продуктивності, мінімізації енергії.

Рассматриваются проблемы проектирования минимальная энергия управления для одного класса динамических систем Такаги-Сугено в непрерывном времени. Работа направлена на выполнение коммутационного закона для подмодели. Предлагаемый подход основан на принципе оптимальности Беллмана. Сравнение четких переключений и нечетких моделей проводится с целью демонстрации эффективности нечеткой модели для оптимизации индекса энергетической эффективности. Эта игра с четкими и нечеткими частями для оптимизации проблемы обсуждается как будущее актуальное направление исследований. Все результаты проиллюстрированы на примерах.нечетких систем вычислительного интеллекта .
Ключевые слова: управление переключением, Такаги-Сугено модели, нечеткая динамическая система, индекс производительности, минимизации энергии.

This paper deals with the problem of designing minimum-energy control for a class of Takagi-Sugeno continuous-time dynamical systems. The work is focused on fulfillment of the switching law for submodels. The proposed approach is based on Bellman's principle of optimality. The comparison of crisp switching and fuzzy models is conducted to demonstrate the effectiveness of the fuzzy model to optimize the energy efficiency index. The different combination of crisp and fuzzy parts for the optimization problem is discussed as future topical trends of studies. All results are illustrated with examples.
Keywords: switching control, Takagi-Sugeno model, fuzzy dynamic system, energy efficiency index, energy minimization.

Introduction. Implementation of the principles of energy management at the enterprises of Ukraine is a key element of the national program to improve efficiency of the energy industry.

The main elements that determine the efficiency of the energy management system are the institutional arrangements, facilities, information technology, the availability of necessary resources. Using of modern project management methodologies in the implementation of the energy management system caused by the innovative nature of the technical solutions, the existing time and financial constraints, high risk, and for quality requirements of the project [1, 2]. With the implementation of changes in the energy sector at the regional level, there is a need for an integrated approach to management, based on a multi-project and program management.

In order to formalize management processes in the energy management are appropriate using of standard 50001:2012 ISO "energy management systems. Requirements with guidance for use." [3]. Using of this standard will integrate the principles of energy management as the management of the organization and the project management, the implementation of which the organization is involved.

In spite of the fact that energy management technical aspects (energy audit, application of energy saving technologies et cetera) are underlaid, efficiency of management projects in energy is determined efficiency of management resources. With the purpose of providing of viability of projects on the initial stage of planning it is necessary to conduct the estimation of resource realizability of project, expose requirements both to to material and technical resources and to the command of project (high-quality and quantitative composition of command, necessary jurisdictions, functions of necessity and presence of resources in a project). In the case of multiproject management and management the brief-case of projects it is necessary to provide optimum allocation of resources within the framework of pool of resources of organization with the purpose of decline of risk of origin of resource conflicts.

During realization of project a requirement in the resources of certain kind is not permanent a size, but changes during time.

Let x – is a resource of project, u – the facilities, necessary for functioning of this resource (expenses on the use, energy consumption et cetera for financial resources and labour costs for human capitals).

Then time-history of this resource can be described differential equalization:

$$\frac{dx}{dt} = f(x,u) ,$$

where u is managing influence (expenses, electric power et cetera) which needs to be minimized for period of time of implementation of project (from 0 to t).

As a system is difficult, to describe it one equalization is not always possible, and depending on the state of kh (more precisely, from his place on a phase plane), different equalizations are used. Together they form the so-called commuted system (switched system).

One of the modern directions in control system theory is to investigate different complex and hybrid systems, among them so-called switching systems[1.2]. In such a system the dynamic, for instance, of the continuous-time controlled process is described with several differential equations or differential inclusions like

$$\dot{x}(t) \in \{ f_\alpha (x(t), u(t)) \}_{\alpha \in A}, \qquad (1)$$

where $x(t) \in R^n$ is a state system, $u(t) \in R^k$ is a control and $\{ f_\alpha : R^{n+k} \to R^n \}_{\alpha \in A}$ is a set of continuously differentiable functions, parameterized by α in a suitable set A. Such systems are used for description of a variety of applications, including situations where a control is generated among a number of subsystems[6].

The choosing of appropriate model usually depends on a part (or a region) of state space in which an object is staying or a model itself is a control parameter. We consider in this paper the case when a model is depended on a region of state space in which state is staying. In this case we can write $A = A(x)$.

We can also formulate the optimization problem, namely, the minimization of a cost function

$$J = J(x, \dot{x}, u) \qquad (2)$$

with restriction (1) and some initial and/or final states

$$x(t_0), x(t_f) \qquad (3)$$

for finite or infinite time t_f of control.

The main objective of an optimal control is to determine control signals $u(t)$ for open systems or $u(x, \dot{x})$ for closed-loop systems that will cause that a process (plant) will satisfy some physical constraints (1) and at the same time extremize (maximize or minimize) a chosen performance criterion (a performance index or a cost function) (2).Taking into account different types of uncertainly there are many formulations of optimal control problems, where (1),(2) or (3) can be described using statistical or fuzzy models [7, 8].

We consider here the case when model (1) is fuzzy and an performance index is a crisp one. We compare also the switching fuzzy model with the crisp one in the sense of the value of the performance index as well.

Decomposition approach for energy performance index minimization using Bellman optimality principle

As a problem of optimization we consider the energy-optimal control system, namely

$$J = \frac{1}{2} \int_{t_0}^{t_f} u^2 dt , \qquad (4)$$

with the model of region rules

$$\begin{aligned} &R1: if\ x\ is\ \Lambda_1\ then\ \dot{x} = f_1(x,u), \\ &R2: if\ x\ is\ \Lambda_2\ then\ \dot{x} = f_2(x,u), \\ &\ldots \\ &RM: if\ x\ is\ \Lambda_M\ then\ \dot{x} = f_M(x,u) \end{aligned} \qquad (5)$$

where Λ_i are regions in the state space. In the crisp case these regions are crisp sets, in a fuzzy case they are the fuzzy sets. For crisp sets we use following restrictions $\bigcup_{i=1}^{M} \Lambda_i = R^n$, $\Lambda_i \bigcap \Lambda_j = \varnothing, \forall i \neq j$. In the case of the fuzzy model, the membership functions of linguistic terms Λ_i are designed under such limitation of normalization, namely $\sum_{i=1}^{M}\mu_i\left(x\right)=1,\forall x\in R^n$.

Giving the energy-optimal criteria we also base on Bellman principle of optimality [6] according to an optimal sequence of decisions in a multistage decision process, the problem has the property that whatever the initial state and decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decisions. This means that we could rewrite (4) as

$$J=\frac{1}{2}\int_{t_0}^{t_f}u^2dt=J_{\alpha_0}+...+J_{\alpha_H}= \\ =\frac{1}{2}\int_{t_0}^{t_1}u^2dt+\frac{1}{2}\int_{t_1}^{t_2}u^2dt+...+\frac{1}{2}\int_{t_H}^{t_f}u^2dt\,, \quad (6)$$

where each part of performance index is minimized for appropriated model (1) or (5) from $\alpha_j\in A\left(x\right)$. Giving models in each part of control time the optimization problem in this case can be formulated as finding the times from the set $\left\{t_1,\ldots,t_H\right\}$ that jointly minimize the performance index (6) with solving the optimization problem for each model $\alpha_j\in A\left(x\right)$. Such decomposition approach allows considering the whole optimization problem as a set of separate optimization problems with additional transversality conditions between models. These conditions depends on type of solution: is the trajectory inside one time interval $\left[t_i,t_{i+1}\right]$ intersect or does not intersect the border of space appropriate interval Λ_j. Most of authors on the topic of switching control does the strict restrictions and does not consider such intersections [6, 8]. Moreover, there are not separate investigations on energy performance index minimization for switching systems using decomposition approach.

We compare in this paper two switching models, namely the crisp model and fuzzy Takagi-Sugeno one for minimization of functional (2).For the sake of simplicity the decomposition approach is illustrated on examples of first order differential equations.

Energy-optimal control system for switching crisp model

Let us consider a following optimization problem of a crisp switching system (5) that is consisted of models R1 and R2:

$$R1: if\ x\geq 0.5\ then\ \dot{x}=u, \\ R2: if\ x<0.5\ then\ \dot{x}=2u \quad (7)$$

with the performance index (4) and initial and final conditions $x\left(0\right)=2, x\left(2\right)=-2$ correspondingly, where $t_0=0, t_f=2$. The initial value $x\left(0\right)=2$ belongs to condition $x\geq 0.5$ that is why we use model R1 in the first part of time from 0 to $t_1<t_f$. The final value $x\left(2\right)=-2$ belongs to condition $x<0.5$ and we use model R2 the rest part of time from t_1 to 2. That is clear that we have $x\left(t_1\right)=0.5$. According to principle of optimality (6) we can write

$$J=\frac{1}{2}\int_0^2 u^2dt=J_1+J_2=\frac{1}{2}\int_0^{t_1}u^2dt+\frac{1}{2}\int_{t_1}^2 u^2dt\,,$$

with partial optimization problems $J_1=\frac{1}{2}\int_0^{t_1}u^2dt$ for model $f_1\left(x,u\right)=u$ and $J_2=\frac{1}{2}\int_{t_1}^2 u^2dt$ for the model $f_2\left(x,u\right)=2u$. On each piece of trajectory we solve the separate optimization problem J_1 and J_2 with the appropriate model from (7). In such a case the whole optimization problem could be formulated as finding the time $t_1<2$ of the switching of the control. Let us solve this task now.

Case 1. Let $x\geq 0.5$, $t\in\left[0,t_1\right]$, $x\left(0\right)=2, x\left(t_1\right)=0.5$. We solve here the optimization problem of minimization $J_1=\frac{1}{2}\int_0^{t_1}u^2dt$ with the plant model $\dot{x}=u$ using variation approach:

$$J_1=\frac{1}{2}\int_0^{t_1}u^2dt=\frac{1}{2}\int_0^{t_1}\dot{x}^2dt=\int_0^{t_1}V\left(x,\dot{x}\right)dt\,,$$

where

$$V\left(x,\dot{x}\right)=\frac{1}{2}\dot{x}^2. \quad (8)$$

The necessary condition to minimize (8) is the Euler-Lagrange equation [8]

$$\frac{\partial V}{\partial x}-\frac{d}{dt}\frac{\partial V}{\partial\dot{x}}=0 \quad (9)$$

that gives the optimal solution

$$x\left(t\right)=C_1t+C_2\,. \quad (10)$$

From initial and final conditions we can find the constants in (10), the optimal control and the performance index $C_2=2, C_1=-\frac{1.5}{t_1}$, $u\left(t\right)=-\frac{1.5}{t_1}$, $J_1=\frac{1}{2}\int_0^{t_1}\frac{1.5^2}{t_1^2}dt=\frac{1.125}{t_1}$. Here time t_1 is to be found.

Case 2. Let $x<0.5$, $t\in\left[t_2,2\right]$, $x\left(t_1\right)=0.5, x\left(2\right)=-2$. We solve here the optimization problem of minimization $J_1=\frac{1}{2}\int_0^{t_1}u^2dt$ with the plant model $\dot{x}=2u$ using variation approach

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$$J_1 = \frac{1}{2}\int_{t_1}^{2} u^2 dt = \frac{1}{2}\int_{t_1}^{2} 4\dot{x}^2 dt = \int_{t_1}^{2} V\left(x,\dot{x}\right)dt\,,$$

where

$$V\left(x,\dot{x}\right) = 2\dot{x}^2\,. \qquad (11)$$

The necessary condition to minimize (11) is the Euler-Lagrange equation (9) that gives the optimal solution

$$x(t) = C_3 t + C_4\,. \qquad (12)$$

From initial and final conditions we can find the constants in (12), the optimal control and the performance index $C_3 = \frac{2.5}{t_1 - 2}$, $C_4 = -\frac{2t_1 + 1}{t_1 - 2}$, $u(t) = \frac{1.25}{t_1 - 2}$, $J_2 = \frac{1}{2}\int_{t_1}^{2} \frac{1.25^2}{\left(t_1 - 2\right)^2} dt = -\frac{0.78125}{t_1 - 2}$.

Thus, we have the following performance index as a function from t_1

$$J\left(t_1\right) = \frac{1}{2}\int_{0}^{2} u^2 dt = J_1 + J_2 = \frac{1.125}{t_1} - \frac{0.78125}{t_1 - 2}\,,$$

$$t_1 \in \left(0,2\right)\,. \qquad (13)$$

To find the minimal value of the performance index (13) is necessary to solve the equation $\frac{\partial J}{\partial t_1} = 0$. We have for (13) $\frac{0.78125}{\left(t_1 - 2\right)^2} - \frac{1.125}{t_1^2} = 0$. The solutions are $t_1 = \left\{1.\left(09\right), 12.0\right\}$. Because $t_1 \in \left(0,2\right)$ we have the moment of switching $t_1 = 1.\left(09\right)$. The value of the performance index $J = \frac{1.125}{t_1} - \frac{0.78125}{t_1 - 2} \approx 1.891$.

Energy-optimal control system for switching fuzzy model

Let us consider the energy optimization problem of a fuzzy switching system (5) that is consisted of models R1 and R2:

$$\begin{aligned} &R1: if\ x\ is\ L_1\ then\ \dot{x} = u, \\ &R2: if\ x\ is\ L_2\ then\ \dot{x} = 2u \end{aligned} \qquad (14)$$

with the performance index (4) and initial and final values $x(0) = 2, x(2) = -2$.

Let L_1, L_2 be the linguistic variables with membership functions

$$\mu_{L_2}\left(x\right) = \begin{cases} 1, if\ x \le 0, \\ 1 - x, if\ 0 < x < 1, \\ 0, if\ x \ge 1 \end{cases} \qquad (15)$$

According to the principle of optimality (6) we can write

$$\begin{aligned} J &= \frac{1}{2}\int_{0}^{2} u^2 dt = J_1 + J_2 + J_3 = \\ &= \frac{1}{2}\int_{0}^{t_1} u^2 dt + \frac{1}{2}\int_{t_1}^{t_2} u^2 dt + \frac{1}{2}\int_{t_2}^{2} u^2 dt \end{aligned} \qquad (16)$$

with partial optimization problems $J_1 = \frac{1}{2}\int_{0}^{t_1} u^2 dt$ for model $f_1\left(x,u\right) = u$, $J_1 = \frac{1}{2}\int_{t_1}^{t_2} u^2 dt$ for the model $f_2\left(x,u\right) = x \cdot u + \left(1 - x\right) \cdot 2u$ and $J_3 = \frac{1}{2}\int_{t_2}^{2} u^2 dt$ for the model $f_3\left(x,u\right) = 2u$.

The model $f_2\left(x,u\right) = x \cdot u + \left(1 - x\right) \cdot 2u$ arises from Takagi-Sugeno inference engine $f_2(x,u) = \frac{\mu_{L_1}(x) \cdot u + \mu_{L_2}(x) \cdot u}{\mu_{L_1}(x) + \mu_{L_2}(x)}$ with limitation of normalization.

On each piece of the trajectory we solve the separate optimization problem J_1, J_2 and J_3 with the appropriate model from (14). In such a case the whole optimization problem could be formulated as finding the time $t_1 < t_2 < 2$ of switching of the control. Let us solve this task now.

Case 1. Let $x \ge 1, t \in \left[0, t_1\right]$, $x\left(0\right) = 2, x\left(t_1\right) = 1$. We solve here the optimization problem of minimization $J_1 = \frac{1}{2}\int_{0}^{t_1} u^2 dt$ with the plant model $\dot{x} = u$ using variational approach:

$$J_1 = \frac{1}{2}\int_{0}^{t_1} u^2 dt = \frac{1}{2}\int_{0}^{t_1} \dot{x}^2 dt = \int_{0}^{t_1} V\left(x,\dot{x}\right)dt\,,$$

where

$$V\left(x,\dot{x}\right) = \frac{1}{2}\dot{x}^2\,. \qquad (17)$$

The necessary condition to minimize (17) is the Euler-Lagrange equation (9) that gives the optimal solution

$$x\left(t\right) = C_1 t + C_2\,. \qquad (18)$$

From initial and final conditions we can find the constants in (12), the optimal control and the performance index

$$C_2 = 2\,,\ C_1 = -\frac{1}{t_1}\,,\ u\left(t\right) = -\frac{1}{t_1}\,,$$

$$J_1 = \frac{1}{2}\int_{0}^{t_1} \frac{1}{t_1^2} dt = \frac{1}{2t_1}\,.$$

Case 2. Let $0 < x < 1, t \in \left(t_1, t_2\right)$, $x\left(t_1\right) = 1, x\left(t_2\right) = 0$. We solve here the optimization problem of minimization $J_1 = \frac{1}{2}\int_{t_1}^{t_2} u^2 dt$ with plant model $\dot{x} = x \cdot u + \left(1 - x\right) \cdot 2u$ using variation approach:

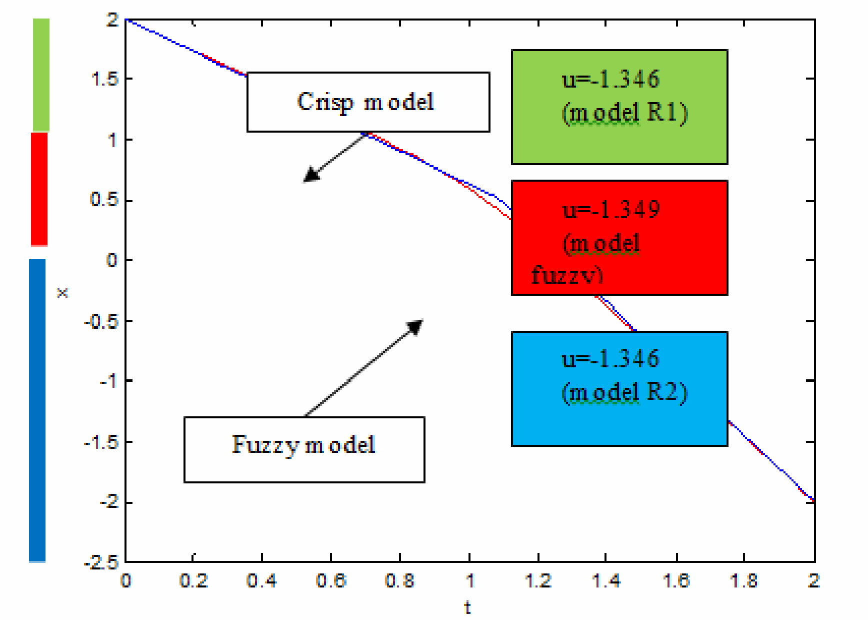

Figure 1 – Optimal trajectory for the fuzzy model (14) and the crisp model (7)

$$J_1 = \frac{1}{2}\int_{t_1}^{t_2} u^2 dt = \frac{1}{2}\int_{t_1}^{t_2} \frac{\dot{x}^2}{(2-x)^2} dt = \int_{t_1}^{t_2} V(x,\dot{x})dt, \quad (19)$$

where $V(x,\dot{x}) = \frac{\dot{x}^2}{2(2-x)^2}$. The necessary condition to minimize (19) is the Euler-Lagrange equation (9) $\dot{x}^2 + 2\ddot{x} - x\cdot\ddot{x} = 0$ whence

$$x(t) = C_3 \cdot e^{C_4 \cdot t} + 2. \quad (20)$$

From initial and final conditions we can find the constants in (20), the optimal control and the performance index:

$$C_4 = \frac{\ln 2}{t_2 - t_1}, \; C_3 = \frac{-2}{e^{\frac{\ln 2}{t_2 - t_1}t_2}}, \; u(t) = -\frac{\ln 2}{t_2 - t_1},$$

$$J_2 = \frac{1}{2}\int_{t_1}^{t_2} \frac{(\ln 2)^2}{(t_2 - t_1)^2} dt = \frac{(\ln 2)^2}{2(t_2 - t_1)}.$$

Case 3. Let $x \le 0, t \in [t_2, 2]$, $x(t_2) = 0, x(2) = -2$. We solve here the optimization problem of minimization $J_3 = \frac{1}{2}\int_{t_2}^{2} u^2 dt$ with the plant model $\dot{x} = 2u$ using variational approach

$$J_1 = \frac{1}{2}\int_{t_2}^{2} u^2 dt = \frac{1}{2}\int_{t_2}^{2} \frac{\dot{x}^2}{4} dt = \int_{t_2}^{2} V(x,\dot{x})dt,$$

where

$$V(x,\dot{x}) = \frac{1}{8}\dot{x}^2. \quad (21)$$

The necessary condition to minimize (21) is the Euler-Lagrange equation (9) that gives the optimal solution

$$x(t) = C_5 t + C_6. \quad (22)$$

From initial and final conditions we can find the constants in (20), the optimal control and the performance index:

$$C_5 = \frac{2}{t_2 - 2}, \; C_6 = \frac{2t_2}{2 - t_2}, \; u(t) = \frac{1}{t_2 - 2},$$

$$J_3 = \frac{1}{2}\int_{t_2}^{2} \frac{1}{(t_2 - 2)^2} dt = \frac{1}{2(2 - t_2)}.$$

Thus, we have the following performance index as a function from t_1, t_2

$$J(t_1, t_2) = J_1 + J_2 + J_3 = \frac{1}{2t_1} + \frac{(\ln 2)^2}{2(t_2 - t_1)} + \frac{1}{2(2 - t_2)}. \quad (23)$$

Let us find the minimal value of the function (23), namely solve the following system: $\frac{\partial J}{\partial t_1} = 0$, $\frac{\partial J}{\partial t_2} = 0$. It leads for (23) to the system of equations

$$\begin{cases} \frac{(\ln 2)^2}{(t_1 - t_2)^2} - \frac{1}{t_1^2} = 0, \\ \frac{1}{(t_2 - 2)^2} - \frac{(\ln 2)^2}{(t_1 - t_2)^2} = 0. \end{cases}$$

Only one solution can be approved according to conditions $t_1, t_2 \in [0, 2], t_1 < t_2$, namely:

$$t_1 = \frac{2}{\ln 2 + 2} \approx 0.743, t_2 = \frac{2(\ln 2 + 1)}{\ln 2 + 2} \approx 1.257.$$

So we have $J(t_1, t_2) \approx 1.813$.

Illustrate the optimal trajectories for crisp and fuzzy models (fig. 1).

Comparison of the crisp and fuzzy models

It is interesting that the optimal fuzzy system model gives less value of the performance index then the crisp model, $J_F < J_C$. Let $t \in [t_1, t_2]$, where the interval $[t_1, t_2]$ is an interval where the fuzzy model works. It is enough to calculate the performance index in interval $[t_1, t_2]$. Really, out of this boundary, namely on the time segments $[0, t_1], [t_2, 2]$ we use the same models and the same values of performance indexes on these segments for crisp and fuzzy models. Let $t_c \in [t_1, t_2]$ is a moment of switching of the crisp model (fig. 2). So we have switching times $t_1 = \dfrac{2}{\ln 2 + 2} \approx 0.743, t_2 = \dfrac{2(\ln 2 + 1)}{\ln 2 + 2} \approx 1.257$ for the fuzzy model and $t_c = 1.(09)$ for crisp one.

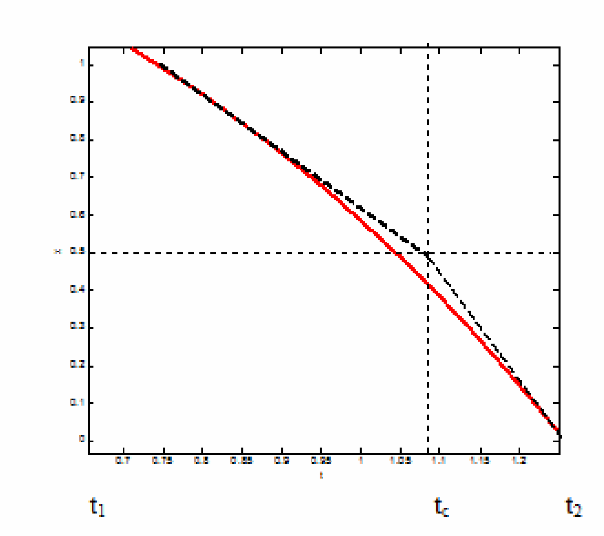

Figure 2 – Difference between the optimal trajectories of crisp and fuzzy models

Let us consider the variation of performance indexes as a function of t_c:

$$\Delta J(t_c) = J_F - J_C =$$

$$= \frac{1}{2}\int_{t_1}^{t_2} \frac{(\ln 2)^2}{(t_2 - t_1)^2} dt - \frac{1}{2}\int_{t_1}^{t_c} \frac{1.5^2}{t_c^2} dt - \frac{1}{2}\int_{t_c}^{t_2} \frac{1.25^2}{(t_c - 2)^2} dt =$$

$$= \frac{(\ln 2)^2}{2(t_2 - t_1)} - \frac{1.5^2}{t_1^2}(t_c - t_1) - \frac{1.25^2}{(t_c - 2)^2}(t_2 - t_c).$$

That is $\Delta J(1.(09)) = -0.444$, so $J_F < J_C$.

Conclusion

In this paper we illustrate the interesting phenomena of advantage of the fuzzy switching model and the crisp one in optimization problem with minimum energy criteria. We show also that such a problem for the switching model can be transformed due to Bellman principle of optimality to the problem of switching time optimization. This article opens also the wide ways for investigation of optimization problems for different combinations of type of optimization elements – crisp, fuzzy Takagi-Sugeno, fuzzy Mamdani, statistical models for representing of performance indexes, plants, initial and transversality conditions so on. We hope to solve these tasks in future step-by-step.

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