The problem of segmentation of the cyclic random process with a segmental structure and the approaches to its solving

I. V. Lytvynenko[*]

Ternopil Ivan Pul'uj National Technical University;
56, Ruska str., Ternopil, Ukraine, 46000

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Abstract

The problem of segmentation (splitting the signal into specific zones, segments) of the cyclic random process with a segmental structure is being described.

There have been introduced several approaches to solving the problem of segmenting the cyclic random process. In particular, they include cyclic and periodic partially stationary random processes. Examples of segmenting simulated and real signals are also provided.

The results can be used in automated processing systems (diagnosis and prognosis) of electrocardiosignals, cyclic economic processes, gas and energy consumption processes, diagnostics of the topographic surface of modern materials.

Keywords: *cyclic random process, segment structure, segmentation.*

Introduction

The existence of various oscillation phenomena and cyclical signals in many areas of science and technology determines the relevance of their modeling, analysis and processing. There arises a problem of their splitting into certain characteristic segments (zones, areas) during the analysis of cyclical signals. In particular, the problems of cycle signal segmentation occur during morphological analysis of cardiac signals [1], analysis of heart rate by cardiointervalogram [1], which is obtained during the periods of rest and exercises. In addition, segmentation problems occur while processing cyclical economic processes [2], gas consumption and energy consumption processes, multiple cracking processes [3], voice signals, etc. Information on cyclical, segmental (zone) structure of the signal let us choose the sampling step [4], assess the rhythmic structure (determine the function of rhythm) of the signal [5], make an analysis, statistical processing and modeling [6–9].

According to studies [8, 9] the cyclical function has a segmental structure where a cycle is the largest segment (the example of a cyclical signal is shown in Figure 1). During this approach the structure and sequence of similar segments (zones) in each cycle of the studied signal is considered to be preserved, because this structure is caused by the deployment in time of the cyclical oscillation phenomena, including the process of heart reduction (Figure 1). Sometimes there can be identified a more detailed segmental structure within each cycle, like in cardiac signals. In medical practice these segments are P, R, S, T zones (Figure 1).

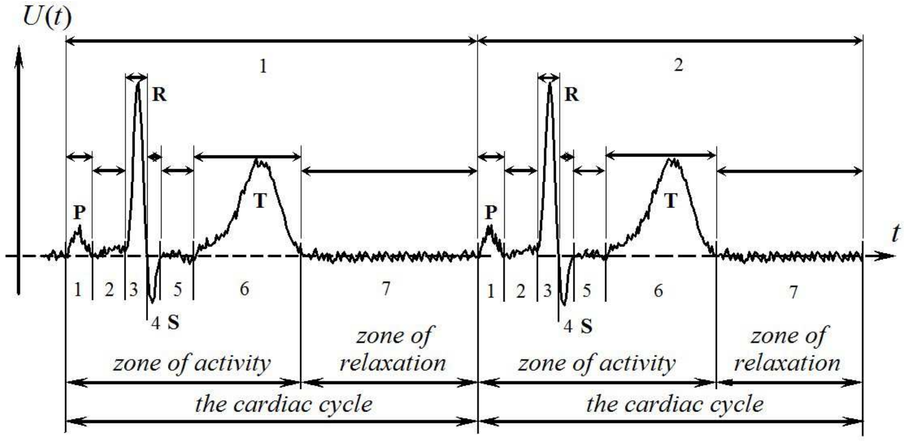
Methods for determining a segmental structure (methods of segmentation) of cyclic signals are based on specific information about the characteristics of certain segments (zones) and they are not universal. That is why it is necessary to use or develop some methods for different types of cyclic signals as there does not exist any unified approach to segmenting cyclic random processes. For example, the work [1] describes the method of electrocardiosignal (ECS) segmentation, which is based on the idea of identifying random process distortion. The works that deal with the problems of the partially stationary random process segmentation [10–12] contain results, which can be used and distributed in segmentation tasks of various cyclic signals for solving the problem of constructing a generalized methodology of cyclic random processes segmentation.

This work deals with formulation and justification of the problem of a cyclic signal segmentation in the frames of the model of a cyclic random process with a segmental structure, and also approaches to its solution.

Mathematical models of cyclic signals

The problem of cyclical signals segmentation can be correctly formulated and solved only on condition that there is a clear mathematical model of a cyclic signal and the concept of a signal segment (cycle, zone) has a clear definition. In order to set the problem of segmenting cyclical signals within the stochastic approach we can define the cyclic random process as a mathematical model of a cyclic signal, and the cyclic random process with a segmental structure (zone time structure). It should be noted that the term

* Corresponding author: iaroslav.lytvynenko@gmail.com



(segments-cycles M = 2, segments-zones N = 7)

Figure 1 – Implementation of the electrocardiosignal with a description of its segmental structure

"zone-temporal structure" can be found in some scientific works due to the specific application area – cardiometry. We apply the term "a segmental structure" in this work as this concept covers and summarizes the concepts of a cycle and zone. At the stage of segmentation, when broken signal segments are not identified as belonging to the cycle or zone, they have a generic name – "a segment".

Definition 1. A separable random process $\xi(\omega,t)$, $\omega\in\Omega,\ t\in\mathbf{R}$ is called a cyclic random process of the continuous argument, if there is a function $T(t,n)$ that satisfies the conditions (1) and (2) of the rhythm function that finite vectors $(\xi(\omega,t_1),\ \xi(\omega,t_2),\dots,\ \xi(\omega,t_k))$ and $(\xi(\omega,t_1+T(t_1,n)),\ \xi(\omega,t_2+T(t_2,n)),\dots,\ \xi(\omega,t_k+T(t_k,n)))$, $n\in\mathbf{Z}$, where $\{t_1,t_2,\dots,t_k\}$ is a set of separability of the process $\xi(\omega,t),\omega\in\Omega,t\in\mathbf{R}$, are stochastically equivalent in a broad sense if integers $k\in\mathbf{N}$ [13, 14].

The function of rhythm $T(t,n)$ determines the variation of time intervals between the single-phase values in its various cycles.

The function $T(t,n)$ must satisfy the following properties:

a) $T(t,n)>0$, if $n>0$ ($T(t,1)<\infty$);

b) $T(t,n)=0$, if $n=0$; (1)

c) $T(t,n)<0$, if $n<0$, $t\in\mathbf{R}$.

For any $t_1\in\mathbf{R}$ and $t_2\in\mathbf{R}$, for which $t_1<t_2$, for the function $T(t,n)$ the following strict inequality is observed:

$$T(t_1,n)+t_1<T(t_2,n)+t_2,\forall n\in\mathbf{Z}\ . \quad (2)$$

The function $T(t,n)$ is the smallest by the absolute value $(|T(t,n)|\le|T_\gamma(t,n)|)$ among all such functions $\{T_\gamma(t,n)\},\gamma\in\mathbf{\Gamma}$, which satisfy (1) and (2).

The cyclic random process of the continuous argument is characterized by the fact that the agreed distribution functions satisfy the following equation:

$$F_{k_\xi}(x_1,\dots,x_k,t_1,\dots,t_k)=$$
$$=F_{k_\xi}(x_1,\dots,x_k,t_1+T(t_1,n),\dots,t_k+T(t_k,n)), \quad (3)$$
$$x_1,\dots,x_k,t_1,\dots,t_k\in\mathbf{R},n\in\mathbf{Z},k\in\mathbf{N}\ .$$

In other words, the cyclic random process is a stochastic process, which distribution functions are invariant by the setoff time arguments to the countable cyclic transformations $\mathbf{\Gamma}=\{T_n(t)=T(t,n)+t,n\in\mathbf{Z}\}$, which are completely determined by the function of rhythm $T(t,n)$. If $T(t,n)=nT,T=const,T>0$, we have a random cyclic process with a stable rhythm or a stochastic T – periodic process. If $T(t,n)\neq nT$, we have a random cyclic process with a variable rhythm.

Before defining a subclass of a cyclic random process – a cyclic random process with a time-zone structure (segmental structure), we give the definition of a random process with time-zone structure introduced in [14].

Definition 2. Let us have the vector N of the random processes defined in the same probability space

$$\Xi_\xi(\omega,t)=\left\{\xi_i(\omega,t),\ i=\overline{1,N},\ \omega\in\Omega,\ t\in\mathbf{R}\right\} \quad (4)$$

and non-random partition $\mathbf{D_R}=\left\{\mathbf{W}_i,i=\overline{1,N}\right\}$ of the domain $\mathbf{R}$, with which indicator functions $\left\{I_{\mathbf{W}_i}(t),\ i=\overline{1,N}\right\}$ are associated and defined according to the following expression

$$I_{\mathbf{W}_i}(t)=\begin{cases}1, & t\in\mathbf{W}_i\\ 0, & t\notin\mathbf{W}_i,\ i=\overline{1,N}.\end{cases} \quad (5)$$

Besides, the temporary subdomains (half-intervals) $\mathbf{W}_i=[t_i,t_{i+1})$ satisfy the following conditions:

$$\bigcup_{i=1}^{N}\mathbf{W}_i=\mathbf{R},\ \mathbf{W}_i\neq\varnothing,\ \mathbf{W}_i\bigcap\mathbf{W}_j=\varnothing,$$
$$\forall i\neq j,\ \ i,j=\overline{1,N}\ . \quad (6)$$

Then a random process that is shown by the construction as follows

$$\xi(\omega,t)=\sum_{i=1}^{N}\xi_i(\omega,t)\,I_{\mathbf{W}_i}(t),\quad \omega\in\Omega,\quad t\in\mathbf{R}, \qquad (7)$$

is called a random process with a segmental (zone) structure of N segments (zones).

It should be noted that a random process (7) with a segmental (zone-temporal) structure can be presented in the following form:

$$\xi(\omega,t)=\sum_{i=1}^{N}\tilde{\xi}_i(\omega,t),\ \omega\in\Omega,\ t\in\mathbf{R}, \qquad (8)$$

where $\left\{\tilde{\xi}_i(\omega,t),\ i=\overline{1,N},\ \omega\in\Omega,\ t\in\mathbf{R}\right\}$ is a set of stochastic processes, equal to:

$$\tilde{\xi}_i(\omega,t)=\xi(\omega,t)\,I_{\mathbf{W}_i}(t),\quad \omega\in\Omega,\quad t\in\mathbf{R}. \qquad (9)$$

The components $\left\{\tilde{\xi}_i(\omega,t),\ i=\overline{1,N},\ \omega\in\Omega,\ t\in\mathbf{R}\right\}$ of construction (8) and the components $\left\{\xi_i(\omega,t),\ i=\overline{1,N},\ \omega\in\Omega,\ t\in\mathbf{R}\right\}$ of construction (7) are intercombined by the following correlations:

$$\tilde{\xi}_i(\omega,t)=\begin{cases}\xi_i(\omega,t),\ t\in\mathbf{W}_i,\\ \quad 0,\ t\notin\mathbf{W}_i.\end{cases}, \qquad (10)$$

$$i=\overline{1,N},\quad \omega\in\Omega,\quad t\in\mathbf{R}$$

or, analogously

$$\tilde{\xi}_i(\omega,t)=\xi_i(\omega,t)\,I_{\mathbf{W}_i}(t), \qquad (11)$$

$$i=\overline{1,N},\quad \omega\in\Omega,\quad t\in\mathbf{R}.$$

That is, corresponding i-th components at the respective i-th sets $\mathbf{R}\setminus\mathbf{W}_i$ are identically equal to zero in construction (8).

The distribution functions of a random process (8) have a structure similar to that of the distribution function of (7):

$$F_{k_\xi}(x_1,\ldots,x_k;t_1,\ldots,t_k)=$$

$$=\sum_{i_1=1}^{N}{}^{(k)}\sum_{i_k=1}^{N}F_{k_{\tilde{\xi}_{i_1}\ldots\tilde{\xi}_{i_k}}}(x_1,\ldots,x_k;t_1,\ldots,t_k)\cdot\prod_{j=1}^{k}I_{\mathbf{W}_{i_j}}(t_j), \qquad (12)$$

$$k\in\mathbf{N},$$

where $\left\{F_{k_{\tilde{\xi}_{i_1}\ldots\tilde{\xi}_{i_k}}}(x_1,\ldots,x_k;t_1,\ldots,t_k)\right\}$ is a set of k-dimensional distribution functions of the vector (8).

Definition 3. A cyclic random process with a zone temporal structure is called a cyclic random process $\{\xi(\omega,t),\omega\in\Omega,t\in\mathbf{R}\}$ with a function of rhythm $T(t,n)$ that can be represented as [8]:

$$\xi(\omega,t)=\sum_{m\in\mathbf{Z}}\xi_m(\omega,t),\ \omega\in\Omega,\ t\in\mathbf{R}, \qquad (13)$$

where $\xi_m(\omega,t)$ corresponds to the m-th cycle of a cyclic random process, which is calculated as:

$$\xi_m(\omega,t)=\xi(\omega,t)\,I_{\mathbf{W}_m}(t),\ m\in\mathbf{Z},\ \omega\in\boldsymbol{\Omega},\ t\in\mathbf{R}; \qquad (14)$$

$I_{\mathbf{W}_m}(t)$ is the indicator function of the m-th cycle equal to

$$I_{\mathbf{W}_m}(t)=\begin{cases}1,\ t\in\mathbf{W}_m,\\ 0,\ t\notin\mathbf{W}_m.\end{cases} \qquad (15)$$

The domains $\mathbf{W}_m$ of the indicator function of the m-th cycle of the process are defined by the half-intervals

$$\mathbf{W}_m=[t_m,t_{m+1}), \qquad (16)$$

where t_m is a moment of the beginning of the m-th cycle of the process.

A cyclic random process can be presented in the following form:

$$\xi(\omega,t)=\sum_{m\in\mathbf{Z}}\sum_{j=1}^{N}\xi_{mj}(\omega,t),\ \omega\in\Omega,\ t\in\mathbf{R}, \qquad (17)$$

where $\xi_{mj}(t),\ t\in\mathbf{W}_{mj}$ is the j-th zone in the *m*-th cycle of a random process equal to:

$$\xi_{mj}(\omega,t)=\xi(\omega,t)\,I_{\mathbf{W}_{mj}}(t)=\xi_m(\omega,t)\,I_{\mathbf{W}_{mj}}(t),$$
$$m\in\mathbf{Z},\ j=\overline{1,N},\ \omega\in\boldsymbol{\Omega},\ t\in\mathbf{R}; \qquad (18)$$

$I_{\mathbf{W}_{mj}}(t)$ is the indicator function of the j-th zone in m-th cycle equal to:

$$I_{\mathbf{W}_{mj}}(t)=\begin{cases}1,\ t\in\mathbf{W}_{mj},\\ 0,\ t\notin\mathbf{W}_{mj}.\end{cases} \qquad (19)$$

The domains $\mathbf{W}_{mj}$ of the indicator function of the j-th zone in the m - th cycle of the process are defined by the half-interval

$$\mathbf{W}_{mj}=\left[t_{m,j},t_{m,j+1}\right), \qquad (20)$$

where $t_{m,j}$ is the reference point of the j-th zone in the m-the cycle of the process.

A random process (14), corresponding to the m-th cycle, is combined with (18), which corresponds to the j-th zone of the cyclic random process, by the following dependence:

$$\xi_m(\omega,t)=\sum_{j=1}^{N}\xi_{mj}(t),\ t\in\mathbf{W}_m,\ \forall m\in\mathbf{Z}. \qquad (21)$$

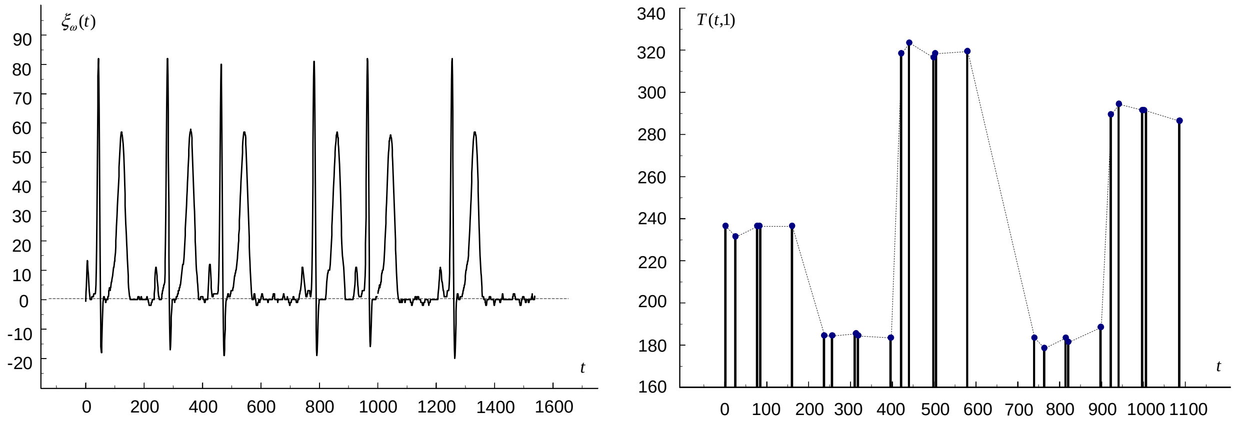
The domains of zones and cycles of the process with a zone cyclic structure are satisfied by the following relation:

$$\mathbf{W}_m=\bigcup_{j=1}^{N}\mathbf{W}_{mj},\ \bigcup_{m\in\mathbf{Z}}\bigcup_{j=1}^{N}\mathbf{W}_{mj}=\mathbf{R},\ \mathbf{W}_{mj}\neq\varnothing,$$
$$\mathbf{W}_{mj_1}\bigcap\mathbf{W}_{mj_2}=\varnothing,\ j_1\neq j_2. \qquad (22)$$

A zone structure of a cyclic random process is given by the set of time points corresponding to the beginning of cyclic process zones:

$$\mathbf{D}_z=\left\{t_{m,j},\ m\in\mathbf{Z},\ j=\overline{1,N}\right\},\ t_m=t_{m,1},\ \forall m\in\mathbf{Z}. \qquad (23)$$

If the cycle is the smallest zone of a cyclic random process, the zone-temporal (segmental) structure may be defined through its cycles: $\mathbf{D}_c=\{t_m,\ m\in\mathbf{Z}\}$.



a) *b)*

a) implementation of the ECS;

b) a discrete function of ECS (partially linear function of the rhythm is marked by the dotted line)

Figure 2 – Implementation of the cyclic random process and its discrete rhythm function

It can be easily shown that a discrete function of rhythm $T\left(t_{m,j},n\right)$, which is enclosed in a continuous one $T\left(t,n\right)$ can be defined by a given set of zones beginnings, namely:

$$T\left(t_{m,j},n\right)=t_{m+n,j}-t_{m,j},\ \forall m,n\in\mathbf{Z}. \qquad (24)$$

If the cycle is the smallest segment (zone) of a cyclic random process, a discrete function of rhythm is defined by the points of the beginnings of random process cycles $\xi\left(\omega,t\right)$:

$$T\left(t_m,n\right)=t_{m+n}-t_m,\ \forall m,n\in\mathbf{Z}. \qquad (25)$$

Thus, the information about beginnings of cyclic process segments (zones) enables us to identify its discrete rhythmic structure, the information which is contained in a discrete function of rhythm $T\left(t_{m,j},n\right)$, which is enclosed in a continuous function of rhythm $T\left(t,n\right)$ of a cyclic random process of a continuous argument. The example of electrocardiosignal and its rhythm functions implementation are shown in Figure 2.

Let us define two important subclasses of a cyclic random process with zone-temporal structure – cyclical and periodic partially stationary stochastic processes.

Definition 4. A cyclic random process with a rhythm function $T\left(t,n\right)$ is called a cyclic partially stationary or a cyclic random process with distortions if it can be given in the following form

$$\xi\left(\omega,t\right)=\sum_{j=1}^{N}\xi_j\left(\omega,t\right)I_{\mathbf{W}_j}\left(t\right)= \\ =\sum_{m\in\mathbf{Z}}\sum_{j=1}^{N}\xi_j\left(\omega,t\right)I_{\mathbf{W}_{mj}}\left(t\right),\ \ \omega\in\Omega,\ \ t\in\mathbf{R}, \qquad (26)$$

where $\left\{\xi_j\left(\omega,t\right),\ \ j=\overline{1,N}\right\}$ is a vector of stationary and stationary associated random processes, a set of indicator functions $\left\{I_{\mathbf{W}_j}\left(t\right),\ \ j=\overline{1,N}\right\}$ is a vector of numerical cyclical functions with the same rhythm functions equal to the function of rhythm $T\left(t,n\right)$ of a cyclic process, i.e.

$$I_{\mathbf{W}_j}\left(t\right)=I_{\mathbf{W}_j}\left(t+T\left(t,n\right)\right),\ I_{\mathbf{W}_j}\left(t\right)=\sum_{m\in\mathbf{Z}}I_{\mathbf{W}_{mj}}\left(t\right),$$

$$\mathbf{W}_j=\bigcup_{m\in\mathbf{Z}}\mathbf{W}_{mj},\ j=\overline{1,N},\ n\in\mathbf{Z},\ t\in\mathbf{R}. \qquad (27)$$

If $T\left(t,n\right)=nT,T=idem$, we have a stochastically T-periodic partially stationary random process or a stochastically T-periodic process with distortions. In this case, indicator functions $\left\{I_{\mathbf{W}_j}\left(t\right),\ \ j=\overline{1,N}\right\}$ are periodic deterministic functions with the period $T>0$ in a structure (26), i.e.

$$I_{\mathbf{W}_j}\left(t\right)=I_{\mathbf{W}_j}\left(t+nT\right),\ j=\overline{1,N},\ n\in\mathbf{Z},\ t\in\mathbf{R}. \qquad (28)$$

This class of processes is characterized by the fact that it combines the properties of stationary random processes (probabilistic characteristics do not change at certain intervals of a parametric set by changing the origin of the process) and the property of nonstationarity (the random process is generally nonstationary, or rather cyclical during the whole time domain).

The problem of segmenting the cyclic random process with a segmental structure

Based on the mentioned mathematical models of cyclical random processes with zone time structure we can formulate a general problem of segmentation of these processes.

The problem of segmenting a cyclic random process consists in calculating an unknown set $\mathbf{D}_z=\left\{t_{m,j},m\in\mathbf{Z},\,j=\overline{1,N}\right\}$, that is, a set of time reference points of j segments-zones in their respective m segments-cycles of a cyclic random process, or calculating partition of the domain of the cyclic random process $\mathbf{W} = \left\{ \mathbf{W}_{m,j}, m \in \mathbf{Z}, j = \overline{1,N} \right\}$. It should be noted that splitting into segments is not enough for the problem of segmentation. It is necessary that there were fulfilled the conditions of isomorphism relatively the order of reference points, which correspond to the segments, and the equality of attributes of segments reference points [5].

$$\begin{gathered} 1)\ t_{m,j} \leftrightarrow t_{m+1,j}, \ldots; t_{m,j+1} > t_{m,j}, \\ t \in \mathbf{W}, m \in \mathbf{Z}, j = \overline{1,N}; \\ 2)\ p(\xi_\omega(t_{m,j})) = p(\xi_\omega(t_{m,j+1})) \to \mathbf{A}, \\ t \in \mathbf{W}, \omega \in \Omega, m \in \mathbf{Z}, j = \overline{1,N}, \end{gathered} \tag{29}$$

where $\mathbf{A}$ is a set of attributes, for example, the attribute is the equality of values of all single-phase reference points, that the following condition is fulfilled $f(t) = f(t + T(t,n)), t \in \mathbf{W}, m \in \mathbf{Z}$ that the attribute is the equality of values $p(f(t)) = f(t)$.

Approaches to the problem solving of segmenting the cyclic random process with a segmental structure

In general, there isn't a single method (or methodology) for solving the problem of cyclic random processes segmentation. It is difficult to solve this problem because it should be clarified, specified a priori information on the probability characteristics of the random cyclic process in each case, in particular, informed about the features the probabilistic characteristics of the process at reference points, which correspond to the beginnings of its segments. Let us consider the approaches to formulating the methods of cyclic random processes segmentation.

The approach to cyclic random process segmentation (stochastic periodic process with zone temporal structure) including partially stationary process.

It should be noted that in case of stochastic periodic process segmentation with the zone time structure, including partially stationary process, it is enough to single out segments-zones only within its m cycle, and all the other segments in other cycles can be calculated if we know the time period T using the following formula:

$$t_{m+n,j} = t_{m,j} + nT, \forall n \in \mathbf{Z}, T > 0,\ j = \overline{1,N}. \tag{30}$$

The period T can be calculated by the following formula

$$T = t_{m,N+1} - t_{m,1}. \tag{31}$$

The approach to segmenting a cyclic random process with a zone temporal structure

When segmenting a cyclic random process with a segmental structure, splitting its one cycle into segments-zones is insufficient because the equality

$$t_{m+n,j} = t_{m,j} + T\left(t_{m,j}, n\right), \forall n \in \mathbf{Z}, T > 0, j = \overline{1,N}, \tag{32}$$

arising out of the equality (24) cannot be applied for calculating the time reference points of segments-zones in other cycles of a random process, as the function of rhythm $T\left(t_{m,j}, n\right)$ is usually unknown during the process of segmentation.

One of the possible approaches to addressing the cyclic random process segmentation is the approach developed in manuscript [1], which is dedicated to segmentation and electrocardiosignal. This work deals with the existence of stationary segments-zones in each cycle of the studied signal using nonparametric statistics (33), which responds to the time points of abrupt change of probability characteristics (expectation). There was made prior segmentation and time reference points of segments-zones were specified:

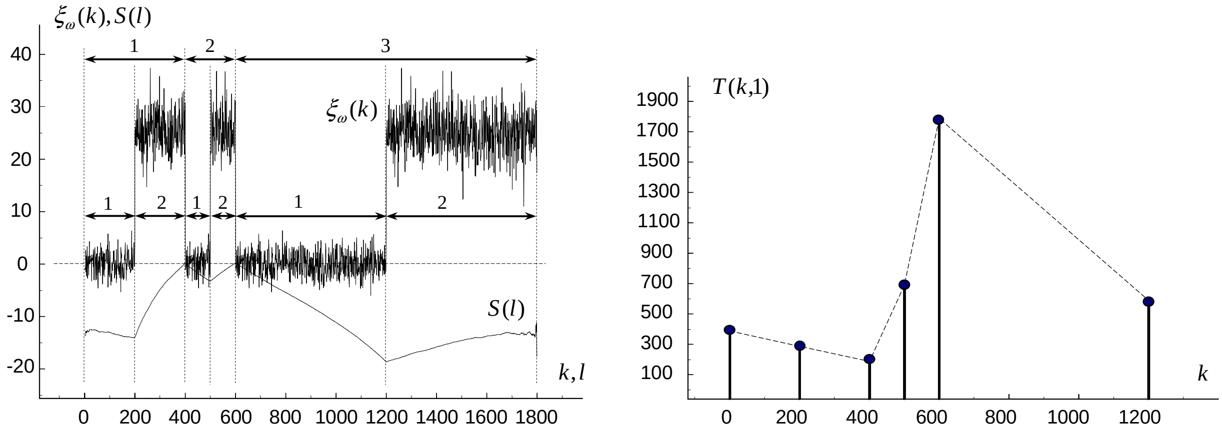
$$S(l) = \frac{1}{l}\sum_{k=1}^{l} \xi_\omega(k) - \frac{1}{K-l}\sum_{k=l+1}^{K} \xi_\omega(k), \quad l = \overline{1, K-1},\ k = \overline{1,K}, \tag{33}$$

where $S(l)$ is the value of statistics at a discrete l time point; K is the number of reference points of the registered signal implementation; $\xi_\omega(k)$ is the value of a discrete signal implementation at k time point.

For testing this approach there were made statistical experiments of segmenting cyclical partially stationary random processes with a cyclic structure. Based on the statistics (33) we have received information on the reference points of segments-cycles and segments-zones, which is used to evaluate the functions of rhythm. The results of this segmentation are shown in Figures 3–5.

Figure 3a shows the statistics response $S(l)$ to sudden changes of the expected value (time points that correspond to the limits of segments-zones of this process). The extremes of statistics $S(l)$ are used for calculating a plurality of time points $\mathbf{D}_z = \left\{ t_{m,j}, m = \overline{1,3},\ j = \overline{1,2} \right\}$ which correspond to beginnings of j-zones in respective m cycles of a cyclical partially stationary random process. Having received information on the segmental structure (zone – temporal structure) $\mathbf{D}_z = \left\{ t_{m,j}, m = \overline{1,3},\ j = \overline{1,2} \right\}$, we define the function of the $T\left(t,1\right)$ based on the correlations (24), (25) and (31) (Fig. 3b).

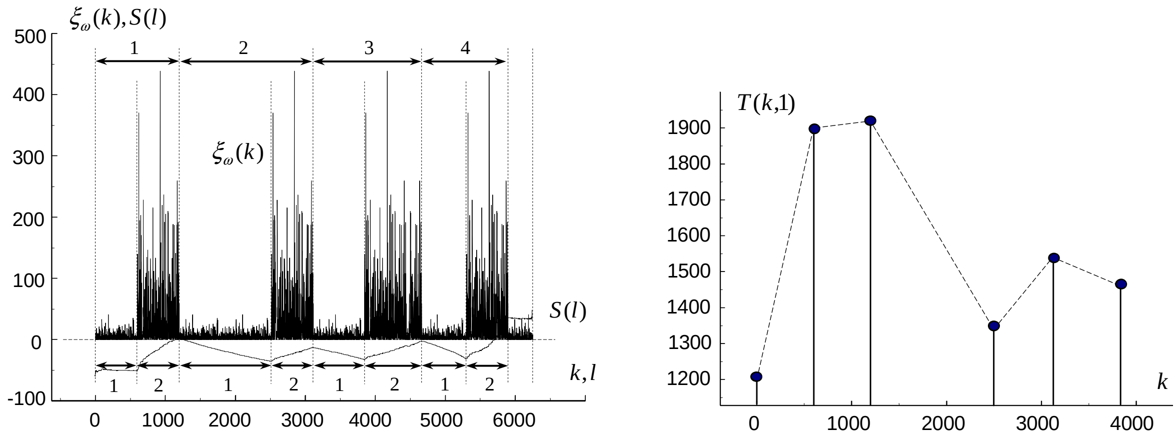
Figure 4 presents the result of segmenting a cyclic partially stationary stochastic process with stationary segments at the cycle. In this case, the defined segmental set $\mathbf{D}_z = \left\{ t_{m,j}, m = \overline{1,4},\ j = \overline{1,2} \right\}$ of time points that correspond to the beginnings of j segments-zones in their respective m segments-cycles of a cyclic partially stationary stochastic process contains 4 cycles with two zones in each cycle. This approach can be used for segmentation, e.g. a speech signal with its mandatory pre-treatment.



a) *b)*

a) implementation of the cyclic partially stationary random process (cycles M=3, zones N=2) and its statistics (the change of the mathematical expectation); b) the discrete rhythm function of the cyclic partially stationary random process has been determined (the partially linear function of the rhythm is marked by the dotted line)

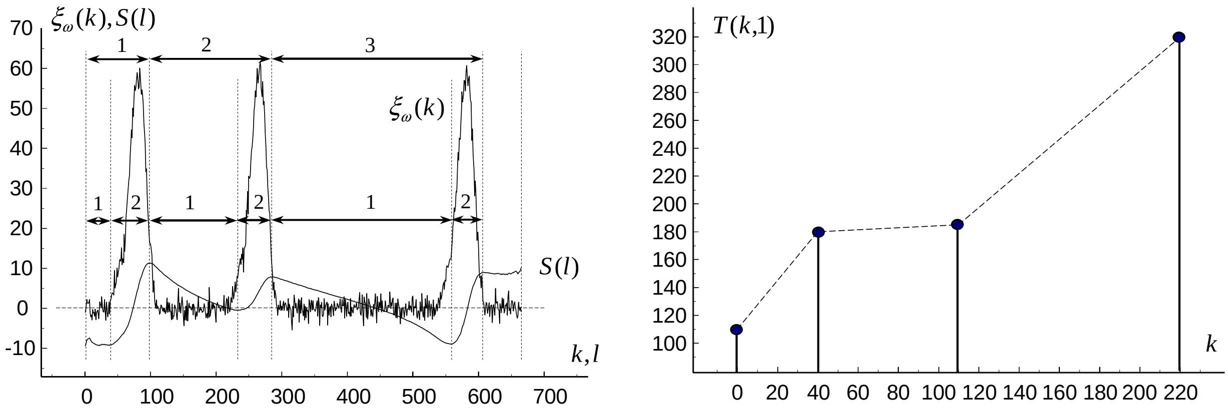
Figure 3 – Cyclic partially stationary random process



a) *b)*

a) implementation of the cyclic partially stationary random process (cycles M=4, zones N=2) and its statistics (the change of dispersion); b) the rhythm function of the cyclic partially stationary random process has been determined (the partially linear function of the rhythm is marked by the dotted line)

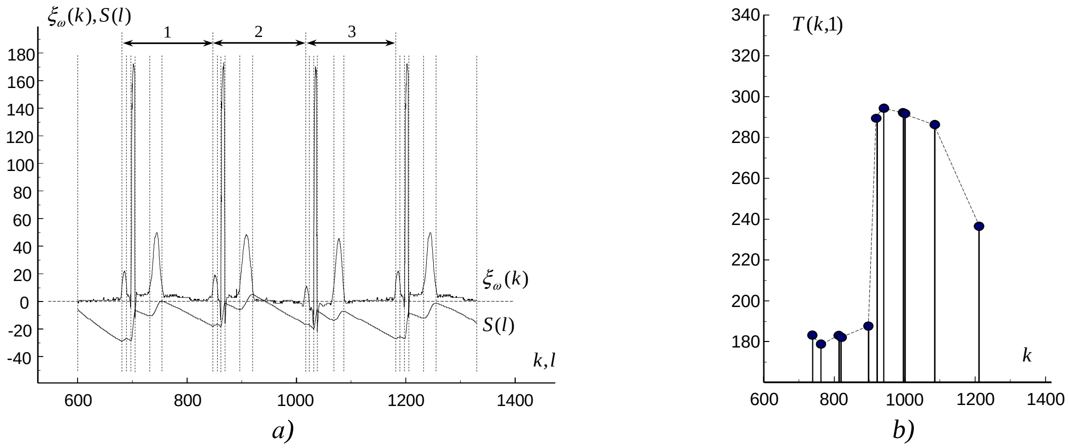
Figure 4 – Cyclic partially stationary random process



a) *b)*

a) implementation of the cyclic random process with stationary segments (cycles M=3, zones N=2) and its statistics (the change of the mathematical expectation); b) the rhythm function of the cyclic random process with stationary segments has been determined (the continuous function of the rhythm is marked by the dotted line)

Figure 5 – Cyclic random process with stationary segments



a) a fragment of implementation of the cyclic random process (a fragment of cycle M=3, zone N=2) and its statistics (the change of the mathematical expectation); b) the rhythm function of the cyclic random process has been determined (the partially linear function of the rhythm is marked by the dotted line)

Figure 6 – Cyclic random process (electrocardiosignal)

Figure 5 presents the result of segmenting a cyclic partially stationary stochastic process. As a result of applying the statistics there were defined a segmental structure $\mathbf{D}_z = \left\{ t_{m,j}, m = \overline{1,3}, j = \overline{1,2} \right\}$, a set of time reference points that correspond to the beginnings of j segments-zones in their respective m segments-cycles and a function of rhythm. The studied random process, the implementation of which is shown in Figure 5, is a non-stationary random process with stationary segments corresponding to the first segment-zone in each cycle. The presence of such stationary segments allows one to apply this approach and to identify the segmental structure as it is shown in [1].

Figure 6 shows the result of segmenting a cyclic random process with stationary segments of an electrocardiosignal. Figure 6 does not show segments-zones, only fragments of the cycles.

The above methods and approaches to segmentation of different cyclic random processes with a segmental structure do not solve all possible problems of cyclic random processes segmentation, and therefore the creation of a generalized methodology of cyclic random processes segmentation remains open.

Conclusion

The problem of segmenting cyclic random processes with segmental structure is formulated in the work. It is shown that the problem of segmentation is solved by applying the known methods of theory of searching distortions of random processes in the case of partially stationary cyclic stochastic process.

The problem of segmenting a cyclic random process with the zone temporal structure, in particular, partially stationary process with a stable rhythm (it can be regarded as a stochastically periodic process with a segmental structure) comes to the allocation of some segments-zones only within one cycle, all the other segments-zones in the cycles can be calculated based on the period.

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Проблема сегментації циклічного випадкового процесу із сегментною структурою та підходи до її вирішення

Я.В. Литвиненко

Тернопільський національний технічний університет імені Івана Пулюя; вул. Руська 56, м. Тернопіль, Україна, 46000

Розглядається проблема сегментації (розбиття сигналу на характерні ділянки, сегменти) циклічного випадкового процесу із сегментною структурою.

Запропоновано ряд підходів до вирішення проблеми сегментації циклічного випадкового процесу, зокрема, циклічного та періодичного кусково-стаціонарних випадкових процесів. Наведено приклади сегментації змодельованих та реальних сигналів.

Результати можуть бути використані в автоматизованих системах обробки (діагностики та прогнозу) електрокардіосигналів, циклічних економічних процесах, процесах газоспоживання та енергоспоживання, процесах діагностики поверхні рельєфоутворень сучасних матеріалів.

Ключові слова: *зонна часова структура, сегментація, циклічний випадковий процес.*