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Application of Hofer's formula in thermal hydraulic calculations of oil transportation systems

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Abstract. Hydrodynamic calculations are a mandatory procedure when designing new and upgrading and operating existing oil transportation systems, which are performed using computer methods. The implementation of new calculation technologies creates conditions for the use of more accurate mathematical models to describe the hydrodynamic processes that accompany the movement of liquid hydrocarbons in a pipeline. For this purpose, based on theoretical studies and mathematical modelling, the article proposed an improved method of thermal-hydraulic calculation of a pipeline for pumping oil and oil products, which takes into account the non-isothermal regime caused by the difference between the product temperature at the beginning of the pipeline and the ground temperature and the release of frictional heat of the flow. The method is based on the application of the universal Hofer's formula for determining the hydraulic drag coefficient in three friction zones of the turbulent regime, involves finding the variable regime coefficients in the Leibenson mathematical model in each section of the pipeline, and applies an integral method for calculating thermal and hydraulic energy losses during the transportation of oil and oil products. The method is suitable for creating computational algorithms and computer programs for design and operational calculations of oil and oil products transportation by pipelines without preheating the products and in the case of using special technologies that involve their preheating. The method was tested by performing multivariate calculations and analysing the results. The analytical dependences of the Leibenzon's model mode coefficients on the Reynolds number and relative surface roughness of the pipe were obtained, which can be used in the thermal-hydraulic calculations of pipelines of all standard diameters at different values of surface roughness

Keywords: oil pipeline; oil product pipeline; hydraulic resistance coefficient; Leibenzon's formula; pressure loss due to friction; non-isothermal flow; flow friction heat

Introduction

The issues of high-quality design and energy-efficient operation of main pipelines and oil product pipelines play an important role in ensuring the energy security of any country, including Ukraine. In the design and operation of main pipelines for the transportation of oil and oil products, i.e. liquid hydrocarbons, one of the main computational operations is hydraulic calculation. The results of hydraulic calculations are used to select pumping equipment for pumping stations, determine the throughput capacity and energy efficiency of the pipeline. A number of works by both Ukrainian and foreign scientists have been devoted to the methods of hydraulic calculation of pipelines. All of them

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use the Darcy-Weisbach equation to calculate pressure losses from friction. Differences in approaches lie in different methods of determining the hydraulic resistance coefficient and different ways of taking into account the physical properties of the medium transported by the pipeline.

In most cases, the existing methods of hydraulic calculation of main pipelines, in the absence of special technologies, do not take into account the processes of heat generation during the movement of fluid and its exchange with the environment in the pipeline-soil system, assuming the operation of such oil transportation systems to be isothermal. In real pipelines, the conditions of isothermal hydrocarbon transportation are rarely met. The product that is pumped into the pipeline usually has a temperature that differs from the ground temperature at the depth of the pipe. In addition, the movement of a real fluid, characterised by viscosity, is accompanied by the generation of frictional heat, which changes the operating temperature and, consequently, the pressure loss from friction in the pipeline. The effect of non-isothermicity is much greater in "hot pipelines", which use a special technology for pumping heated high-viscosity hydrocarbons. The hydraulic calculation of such pipelines also needs to take into account the frictional heat of the flow. This factor of influence on the operating parameters of the oil transportation system is often neglected, as noted by H. Li *et al*. (2024). Non-isothermal pumping is characterised by a change in the thermophysical properties of the transported product due to temperature changes along the pipeline. This temperature change can be described by the heat balance equation, which must take into account the internal frictional heat of the flow. This opinion is shared by a number of authors of other scientific papers, such as U.K. Zhapbasbayev *et al*. (2021) and E. Liu *et al*. (2022).

The current Ukrainian standards for the technological design of main oil pipelines recommend a method of hydraulic calculation of the pipeline based on the steady-state isothermal operation of the pipeline, where the temperature, density, volume flow rate and viscosity of the transported product are constant along the length. Regulatory documents and existing methodologies offer dozens of formulas for the hydraulic resistance coefficient, depending on the mode of product movement - laminar, transient or turbulent. The current standards for the technological design of oil pipelines provide for the use of four formulas for the hydraulic drag coefficient, which do not fit well with the recommended Reynolds limit numbers (DNTD 2-86, 1986).

The formulas for the Reynolds limit numbers proposed by different authors give fundamentally different results. All this makes it difficult to create computational algorithms and computer programmes to determine the capacity and energy efficiency of oil and oil product pipelines. In the world practice, hydraulic calculations of pipelines are based on universal mathematical models that provide reliable results for determining the hydraulic drag coefficient for all three zones of the turbulent regime. An analysis of numerous publications by scientists has shown a unanimous opinion that the Colebrook and White formula provides the greatest accuracy in determining the hydraulic drag coefficient. Since this formula does not give an explicit solution, scientists, in particular F.L. Kaseng *et al*. (2020), S.L. Tolentino & O.G. Campos (2023) and M. López-Silva *et al*. (2024), have been working on methods to create reliable approximations of it for a long time. The approximation formulas proposed by scientists have different complexity of expressions and provide unequal compliance with the basic formula of Colebrook and White. P. Benner *et al*. (2018) and C. Allen *et al.* (2021) noted that for Reynolds numbers and relative roughness values that correspond to the conditions of pipeline hydrocarbon transport, the most successful approximation of the universal Colebrook and White formula is the Hofer's formula. C.-A. Safta *et al*. (2021) confirmed the exceptional impact of pressure losses on the efficiency of hydropower systems. They investigated the scope of application of various formulas for the hydraulic resistance coefficient, including the Stokes, Blasius, and Colebrook-White formulas for determining pressure losses from friction in pipelines operating at Reynolds numbers up to $Re = 6{,}000$. In Ukrainian practice, the universal Colebrook and White formula and its successful approximations are practically not used. As an exception, the work of M.D. Serediuk & N.V. Motruk (2024) should be noted, which proved the feasibility of using the Hofer's formula in hydraulic calculations of high, medium and low-pressure gas networks.

Thus, despite a significant number of publications, the issues of a reasonable choice of mathematical models for the hydraulic resistance coefficient, taking into account the effect of flow friction heat on the thermal and hydraulic parameters of main oil and oil product pipelines have not been fully clarified and require additional research. The proposed study is devoted to improving the methods of design and operational calculations of pipelines for the transportation of oil and its products using traditional and special technologies. The aim of the study was to develop a method for the hydraulic calculation of a pipeline for the transportation of liquid hydrocarbons (oil and oil products), based on the application of the universal Hofer's formula for the hydraulic resistance coefficient and taking into account the non-isothermal factor caused by the difference between the initial temperature of the product and the ground temperature and the generation of frictional heat in the flow. The research objective was achieved through the following tasks:

- development of a universal method for hydraulic calculation of the pipeline, which can be used both for traditional transportation technology without heating and for special technology of pumping preheated liquid hydrocarbons;
- implementation of the proposed method in a computer programme;
- based on the results of hydrodynamic calculations, obtaining analytical expressions for the mode coefficients in the generalised Leibenzon's formula using the universal Hofer's formula for the hydraulic resistance coefficient.

Materials and Methods

When developing the pipeline hydraulic calculation method, it was assumed that hydrocarbons are pumped in a temperature range that ensures their Newtonian properties with sufficient accuracy for practical calculations. The first block of the method involved determining the total heat transfer coefficient from the product to the environment. The value of the total heat transfer coefficient K for underground pipeline installation mainly depends on the external heat transfer coefficient. In turn, the value of this coefficient is largely determined by the value of the soil thermal conductivity coefficient, which varies seasonally. The internal coefficient of heat transfer from the product to the inner surface of the pipe was calculated using the criterion equations of Reynolds, Nusselt, Prandtl and Grashof, depending on the mode of product movement in the pipeline. The following mathematical models were used to reduce the physical properties of the transported product to the calculated temperature t, °C. For density, kg/m^3:

$$\rho = \rho_{20} - \xi(t - 20), \tag{1}$$

where ρ_{20} is density of the product at a temperature of 20 °C; ξ – is a temperature correction. For kinematic viscosity, m^2/s:

$$\nu = a_1 + a_2 \times t + a_3 \times t^2 + a_4 \times t^3, \tag{2}$$

where a_1, a_2, a_3, a_4 are the coefficients of the mathematical model of the product's kinematic viscosity dependence on temperature. For heat capacity, J/(kg·°C):

$$c = \frac{31.56}{\sqrt{\rho_{20}}}(1{,}687 + 3.39 \times t). \tag{3}$$

For thermal conductivity coefficient, W/(m · °C):

$$\lambda_{hr} = \frac{137}{\rho_{20}}(1 - 0.00054 \times t). \tag{4}$$

The developed algorithm provides for the implementation of the method of successive approximations by the average flow temperature of the transported product and the average temperature of the pipe wall when determining the internal heat transfer coefficient. To take into account the effect of non-isothermicity caused by both the difference between the initial temperature of the transported product and the ground temperature as well as the influence of frictional heat of the flow, it is necessary to perform joint thermal and hydraulic calculations of the pipeline. The heat balance equation, which takes into account the frictional heat of the flow, for a pipeline section of length , is presented in the form:

$$\frac{K\pi D}{\rho Qc}dx = -\frac{dt}{t - t_o - \phi\nu^m}, \tag{5}$$

where ϕ is a set of parameters, the value of which depends on the mode of product movement;

$$\phi = \beta\frac{\rho Q^{3-m}g}{K\pi D^{6-m}}; \tag{6}$$

$$\beta = \frac{2^{3-2m}A}{\pi^{2-m}g}, \tag{7}$$

D – inner diameter of the pipeline; t_o – ground temperature at the depth of pipeline laying; Q – volumetric flow rate; g – acceleration of gravity; A, m – coefficients of the fluid motion mode in the Leibenzon's mathematical model for the hydraulic resistance coefficient. In hydraulic calculations of pipelines for the transportation of oil and oil products, the generalised Leibenzon's formula for the hydraulic resistance coefficient is often used:

$$\lambda = \frac{A}{Re^m}, \tag{8}$$

where Re – Reynolds number. This simple mathematical model is a convenient tool for solving a number of issues related to the determination of hydrodynamic parameters of complex pipelines under different modes of hydrocarbon pumping. This formula was also used in this study. Its application made it possible to take into account the frictional heat of the flow when calculating the hydraulic resistance coefficient using the Hofer's formula:

$$\lambda = \frac{1}{\left\{2lg\left[\frac{4.518}{Re}lg\left(\frac{Re}{7}\right) + \frac{k_e}{3.71D}\right]\right\}^2}, \tag{9}$$

where k_e – is the absolute equivalent roughness of the pipe's inner surface. Methods of hydraulic calculation of pipelines used in practice provide for the application of specific constant values of the following coefficients, in particular $A = 0.3164$ and $m = 0.25$, which correspond to the empirical Blasius formula for the hydraulic drag coefficient in the zone of hydraulically smooth pipes in the turbulent regime. However, this formula has a limited scope in terms of the Reynolds number (DNTD 2-86, 1986). This makes it necessary to use other mathematical models for hydraulic calculations of pipelines, which include not only the Reynolds number, but also the value of pipe roughness and do not contain mode coefficients A and m.

Thus, in order to use formula (5) for the thermal calculation of the pipeline, taking into account the non-isothermal mode of product pumping, it was necessary to develop a method for finding the values of the mode coefficients A and m in the case of using other, universal formulas for the hydraulic resistance coefficient. The proposed method involves the use of the Hofer's formula (9) for all three friction zones of the turbulent regime. Unlike existing methods of hydraulic calculation of pipelines, when taking into account the non-isothermicity factor, the coefficients of the motion mode A and m were taken as variable values rather than constant ones. To obtain the corresponding dependences, formula (8) was differentiated by the Reynolds number and the differential $\frac{d\lambda}{dRe}$ from the Hofer's formula (9) was substituted into the obtained expression. As a result, the following formula for the mode coefficient is obtained m (Serediuk & Motruk, 2024):

$$m = \frac{7.849\left[lg\left(\frac{Re}{7}\right) - 0.4343\right]\sqrt{\lambda}}{\left[4.518\,lg\left(\frac{Re}{7}\right) + Re\frac{k_e}{3.71 \times D}\right]}. \tag{10}$$

The value of the mode coefficient A for this method was found as follows:

$$A = \lambda \times Re^m. \quad (11)$$

As shown by formulas (10) and (11), the variable values of the transported product flow regime coefficients A and m depend on the Reynolds number and the roughness of the pipeline's internal surface. Generally speaking, in a non-isothermal pipeline, in certain sections of it, in addition to the turbulent pumping mode, a laminar or transient mode may occur. The following approach is proposed for determining the coefficients of the flow regime under these conditions. For the laminar regime, provided that $Re < Re_{kr} = 2{,}041$:

$$m = 1, A = 64; \quad (12)$$

for the transitional regime when applying the formula recommended by the current regulatory document (DNTD 2-86, 1986):

$$\lambda = (0.16\, Re - 13) \times 10^{-4}; \quad (13)$$

for $Re_{kr} < Re < Re_{per}$ the following expression is obtained:

$$m = -1.6 \times 10^{-5} \frac{Re}{\lambda}, \quad (14)$$

where Re_{per} – is the transient Reynolds number that separates the transient and turbulent modes of product flow in the pipeline. The transient Reynolds number divides the scope of formula (13) and Hofer's formula (9), providing the same calculation results when they are combined. This is important when implementing the method of successive approximations when calculating the operating parameters of pipelines. Similarly, the proposed value of the critical Reynolds number Re_{kr} = 2,041 provides an accurate matching of the Stokes formula and formula (13) at the boundary of laminar and transient pumping modes. The value of the transient Reynolds number depends on the value of the relative roughness of the pipe. M.D. Serediuk & N.V. Motruk (2024) obtained the following formula, which describes the above dependence with an approximation reliability of more than 99% for the entire possible range of relative roughness of standard diameter pipes:

$$Re_{per} = 2{,}836 + 5{,}036 \times \varepsilon, \quad (15)$$

where $\varepsilon = \frac{k_e}{D}$ – is the relative roughness of the pipe surface. The above method made it possible to use the following calculation formula for the thermal regime of a non-isothermal pipeline (Serediuk & Pylypiv, 2013):

$$\frac{K\pi DL}{\rho_{ave} Q c_{ave}} = \int_{t_k}^{t_p} \frac{dt}{t - t_o - \phi v^m}, \quad (16)$$

where L – pipeline length; t_p, t_k – product temperature at the beginning and end of the pipeline; *ave* – is an index indicating the average value of the parameter along the pipeline. Equation (16) was solved with respect to the final product temperature t_k. The computational algorithm involved the use of the method of successive approximations and the numerical method of integrating the right-hand side of equation (16). Formula (16) is written in the following form:

$$Shu = I_t, \quad (17)$$

where $Shu = \frac{K\pi DL}{\rho_{ave} Q c_{ave}}$ Shukhov's parameter. For each value of the product temperature t corresponding to a certain cross-section x of the pipeline, the following operations were performed. The local value of the Reynolds number Re in an arbitrary cross-section of a non-isothermal pipeline was determined. The corresponding values of the coefficients of the mode of motion and the values of the complexes of parameters β and ϕ associated with the consideration of the influence of frictional heat of the flow, were calculated. Using the Simpson method, the value of the determined integral I_t at which the value of the final temperature t_k is equal to the Shukhov parameter was numerically determined. After completing the thermal calculation of the pipeline, hydrodynamic calculations were performed. For a non-isothermal pipeline, the expression for the elementary friction head loss over a section of length using the generalised Leibenson mathematical model for the hydraulic drag coefficient is as follows:

$$dh_\tau = \frac{\beta v^m Q^{2-m} dx}{D^{5-m}}. \quad (18)$$

According to the heat balance equation, which takes into account the effect of flow friction, the relationship between the values dx and dt is as follows:

$$dx = \frac{l}{Shu} \times \frac{dt}{t - t_o - \phi v^m}. \quad (19)$$

Taking into account formulas (18) and (19), the equation for the head loss due to friction in a non-isothermal pipeline is written as:

$$h_\tau = \frac{L}{Shu} \int_{t_k}^{t_p} \frac{\beta v^m Q^{2-m} dt}{D^{5-m}(t - t_o - \phi v^m)}, \quad (20)$$

or

$$h_\tau = \frac{L}{I_t} I_h. \quad (21)$$

Using the Simpson method described above, the following integral was calculated numerically I_h, the value of which is proportional to the pressure loss from friction in a non-isothermal pipeline. Based on the above algorithm, a computer program was developed that makes it possible to calculate the final temperature and pressure losses for the non-isothermal technology of pipeline transport of liquid hydrocarbons. In order to test the proposed methodology, multivariate thermal-hydraulic calculations were performed with the following data: length of the pipeline section between oil pumping stations L = 120 km; outer diameter of the pipeline $D_з$ = 0.530 m; inner diameter of the pipeline D = 0.514 m; no thermal insulation; depth of the pipeline axis h_o = 1.5 m; oil temperature at the beginning of the pipeline t_p = 10 °C; ground temperature at the depth of pipeline laying t_o = 1 °C; thermal conductivity of the ground λ_{hr} = 1.0 W/(m · °C); thermal conductivity of the pipe metal

$\lambda_c = 58$ W/(m · °C). Coefficients of the mathematical model of oil viscometricity: $a_1 = 77.6 \times 10^{-6}$; $a_2 = -8.041 \times 10^{-6}$; $a_3 = 0.3674 \times 10^{-6}$; $a_4 = -5.582 \times 10^{-9}$. Oil density at temperature 20 °C $\rho_{20} = 867$ kg/m^3. Absolute equivalent roughness of the inner surface of the pipe $k_e = 1 \times 10^{-4}$ m.

Results and Discussion

The results of thermal and hydraulic calculations based on the developed and existing methods were performed for different degrees of oil pipeline loading and are summarised in Table 1.

Table 1. Results of oil pipeline calculation taking into account frictional heat of oil flow

Parameter	Parameter value at oil consumption, m³/h			
	500	700	900	1,100
Shukhov parameter	1.3289	0.9572	0.7459	0.6111
Oil temperature at the end of the pipeline, °C	4.0	5.6	7.1	8.5
Total heat transfer coefficient from oil to the environment, W/(m²·°C)	1.53	1.54	1.54	1.54
Frictional pressure loss according to the developed method, MPa	1.49	2.61	3.97	5.57
Pressure loss from friction by the method of isothermal pumping, MPa	1.74	3.13	4.86	6.90
The degree of refinement of energy consumption of oil transportation, %	−14.2	−16.6	−18.2	−19.3

Note: initial oil temperature is 10°C
Source: developed by the authors

As can be seen from Table 1, taking into account the non-isothermal factor when transporting medium-viscosity oil through a pipeline makes it possible to significantly refine its hydrodynamic energy consumption. Table 2 contains the results of the thermal-hydraulic calculation of the pipeline, the parameters of which are indicated above, when pumping oil with the same physical properties as in the previous case, but heated to a temperature of 30°C.

Table 2. Results of oil pipeline calculation taking into account frictional heat of oil flow

Parameter	Parameter value for oil consumption, m³/h			
	500	700	900	1,100
Shukhov parameter	1.3289	0.9517	0.7413	0.6071
Oil temperature at the end of the pipeline, °C	1.54	1.54	1.55	1.55
Total heat transfer coefficient from oil to the environment, W/(m²·°C)	9.18	13.10	16.42	19.10
Frictional pressure loss according to the developed method, MPa	1.27	2.25	3.49	4.99
Pressure loss from friction by the method of isothermal pumping, MPa	1.74	3.13	4.86	6.90
The degree of refinement of energy consumption of oil transportation, %	−27.0	−28.2	−28.1	−27.7

Note: initial oil temperature is 30°C
Source: developed by the authors

As noted above, in the case of applying the proposed method of thermal-hydraulic calculation of the pipeline, the mode coefficients are variable parameters, functions of the Reynolds number and relative roughness of the pipe surface. To establish the form of these functions, the developed program performed multivariate calculations and found the values of the mode coefficients for the entire range of possible loading of the oil pipeline, the parameters of which are given above. Based on the results of the calculations, graphical dependences of the values of the mode coefficients *m* and *A* and the Reynolds number were constructed. They were mathematically modelled using the Excel spreadsheet processor. The results are illustrated in Figure 1.

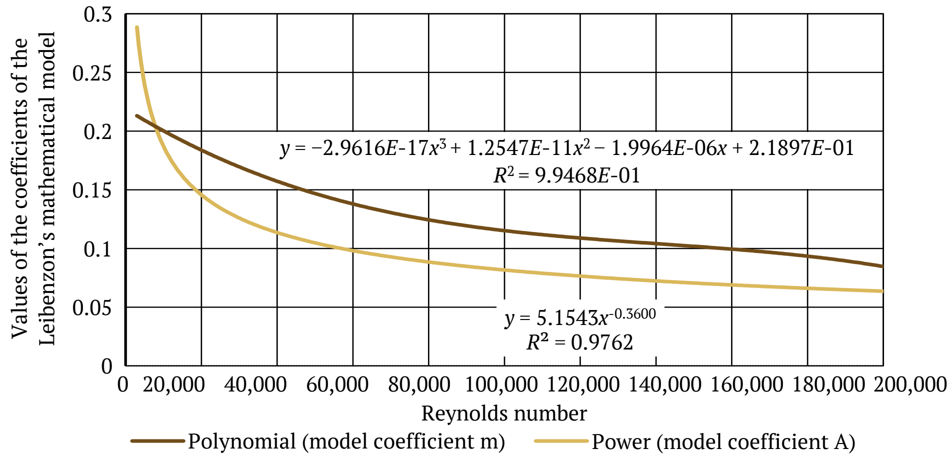

Figure 1. Dependence of the coefficients of the motion mode in the Leibenzon's formula on the Reynolds number for a pipeline with a diameter of 0.514 m and an absolute roughness of 1×10^{-4} m

Source: developed by the authors

In order to transfer the results obtained to pipelines of other diameters with different surface roughness, the Hofer's formula was written in terms of the relative roughness of the pipe ε. The possible range of changes in the diameters of main pipelines and their surface roughness were determined. The absolute equivalent roughness of real pipelines varies from 1×10^{-4} m to 2×10^{-4} m. The standard diameters of pipelines for pumping liquid hydrocarbons vary from DN 100 to DN 1,200. Based on the results of multivariate thermal-hydraulic calculations and their mathematical modelling using the Excel spreadsheet processor, the following generalised dependencies of the flow mode coefficients on the Reynolds number and relative roughness are obtained, which are valid for a pipeline of any diameter. For the coefficient m:

$$m = b_1 + b_2 \times Re + b_3 \times Re^2 + b_4 \times Re^3, \qquad (22)$$

where b_1, b_2, b_3, b_4 are the coefficients of the mathematical model, the values of which depend on the relative roughness of the pipe surface:

$$b_1 = 0.2102 \times \varepsilon^{-0.0048}; \qquad (23)$$

$$b_2 = -1.9048 \times 10^{-6} - 4.8951 \times 10^{-4} \times \varepsilon + 2.4069 \times 10^{-1} \times \varepsilon^2; \qquad (24)$$

$$b_3 = 1.1792 \times 10^{-11} + 4.0277 \times 10^{-9} \times \varepsilon - 2.0944 \times 10^{-6} \times \varepsilon^2; \qquad (25)$$

$$b_4 = -2.7258 \times 10^{-17} - 1.4580 \times 10^{-14} \times \varepsilon + 1.5646 \times \times 10^{-11} \times \varepsilon^2 - 6.3197 \times 10^{-9} \times \varepsilon^3; \qquad (26)$$

for the coefficient A:

$$A = K_a \times Re^{\chi}, \qquad (27)$$

where K_a, χ – are the coefficients of the mathematical model, the values of which depend on the relative roughness of the pipe surface:

$$K_a = 13.764 - 1.5363 \times 10^4 \times \varepsilon + 7.4986 \times 10^6 \times \varepsilon^2; \qquad (28)$$

$$\chi = -0.47346 + 166.10 \times \varepsilon - 57{,}913 \times \varepsilon^2. \qquad (29)$$

The reliability of the approximation of formulas (22-29) exceeds 98%. The proposed methodology, with the reasonable use of the Hofer's formula to determine the hydraulic resistance coefficient of the system and taking into account the non-isothermal flow of the medium transported by main pipelines, as well as the obtained analytical dependences of the mode coefficients included in the generalised Leibenson formula, can be used with a sufficiently high accuracy of the results obtained in the development of computational algorithms for thermal and hydraulic calculations of pipelines of complex configuration, which implement various modes of liquid hydrocarbon pumping with appropriate rheological characteristics.

Progressive technologies for the production and use of alternative gaseous energy carriers, including gas-hydrogen mixtures, hydrogen, biomethane, etc., are being intensively introduced. Despite this, the scope and volume of use of liquid hydrocarbons – oil and oil products – is not decreasing. Pipeline transport plays a crucial role in transporting large volumes of liquid hydrocarbons over long distances. Pipeline transportation of hydrocarbon energy carriers, in particular oil and oil products, is an influential strategic factor in the country's economic stability and energy security, provided that it is operated reliably and safely. In the global practice of hydraulic calculations, different formulas are used for the hydraulic resistance coefficient, taking into account the purpose and geometric characteristics of the pipeline, physical properties of the transported product and pumping volumes.

For example, T. Bekibayev *et al.* (2021) presented the results of studies on the identification of the hydraulic resistance coefficient in main oil pipelines. The authors consider the process of non-isothermal oil pumping, taking into account heat exchange with the environment. The change in oil viscosity along the length of the pipeline, as well as the change in pipe surface roughness during its operation, are taken into account. Based on the data of industrial experiments on existing oil pipelines, a formula for the hydraulic resistance coefficient is proposed for the laminar mode of pumping, which differs by a numerical coefficient from the classical Stokes formula, stating that the use of the Colebrook and White formula for the turbulent mode of oil flow causes certain difficulties in the implementation of hydraulic calculations of pipelines. Therefore, it is advisable to use reliable approximations of this formula. A modified Altschul formula is proposed for determining the hydraulic drag coefficient, which gives results that differ by several percent from the Colebrook values. It should be noted that the adequacy of the Altschul formula was verified based on the results of industrial experiments in a narrow range of Reynolds numbers $Re = (10{,}000 - 30{,}000)$. The actual range of Reynolds numbers in main oil and oil product pipelines is much larger and can be $Re = (3{,}000 - 200{,}000)$. T. Bekibayev *et al.* (2021) proposed an adjusted Stokes formula for the laminar regime, based on the results of industrial experiments at $Re < 2{,}100$:

$$\lambda = \frac{71.6}{Re}. \tag{30}$$

For the turbulent regime of oil pumping through a pipeline at Reynolds numbers, T. Bekibayev *et al.* (2021) confirmed the adequacy of the Altschul formula, which is an approximation of the Colebrook and White formula. N. Luta & N. Antonyuk (2024) recommended using a complex approximation of the Zigrang and Sylvester equation in the turbulent mode of pumping light oil products:

$$\frac{1}{2\sqrt{\lambda}} = -2\,lg\left\{\frac{k_e}{3.7D} - \frac{5.02}{Re}\,lg\left[\frac{k_e}{3.7D} - \frac{5.02}{Re}\,lg\left(\frac{k_e}{3.7D} + \frac{13}{Re}\right)\right]\right\}. \tag{31}$$

M.D. Serediuk & N.V. Motruk (2024) proved the feasibility of using the Hofer's formula for the hydraulic resistance coefficient in the hydraulic calculation of gas distribution networks of all categories of working pressure. A.O. Gallardo *et al.* (2021), Z. Hafsi (2021) and H.M. Benavides-Muñoz (2024), and many others have confirmed that the Colebrook and White formula provides the highest accuracy in determining the hydraulic drag coefficient in turbulent conditions. A.O. Gallardo *et al.* (2021) analysed the scope of application of seven explicit approximations of this formula and proposed their own version for the hydraulic calculation of pipeline water distribution systems. The works by R.T.D.A. Minhoni *et al*. (2020) and L.E. Muzzo *et al.* (2021) compared the explicit approximations of the Colebrook and White equation for determining the hydraulic resistance coefficient proposed by other authors in terms of the accuracy of the calculation results and the efficiency of application.

H.M. Benavides-Muñoz (2024) proposed two new refined modifications of the Churchill's equation to calculate the hydraulic drag coefficient. The reliability and accuracy of the conducted studies are substantiated by comparing them with the results obtained by the Colebrook and White formula. Z. Hafsi (2021), to determine the hydraulic drag coefficient, gave a direct analytical solution to the Colebrook and White equation by decomposing a third-order polynomial using the Cardano method. F.A. Daneshvar *et al*. (2023) proposed a methodology for determining the hydraulic drag coefficient based on the numerical solution of the Colebrook and White equation in the studied range of fluid velocities, commercial diameters, and roughness parameters of GRP pipes. In X. Fang *et al.* (2020) provided an overview of existing correlation models for determining the hydraulic drag coefficient in refrigerant transport systems at supercritical pressures and propose a new friction coefficient model for turbulent flow with high prediction accuracy.

F. Fiorillo *et al*. (2024) proposed a methodology for applying Poiseuille and Darcy-Weisbach laws to describe groundwater flow under laminar and turbulent fluid motion in porous aquifers and their relationship in the transition zone between laminar and turbulent flows. I. Santos-Ruiz *et al*. (2021) proposed a fundamentally different way to improve the accuracy of pipeline hydraulic design under turbulent conditions. Using the Lambert function and nonlinear optimisation methods, the authors processed the results of experimental studies of fluid pumping modes at a pilot pipeline installation. As a result, the authors established the dependence of the hydraulic drag coefficient on the Reynolds number and pipe surface roughness.

G.B. Ferreri (2024) proposed a new approach to estimating the hydraulic drag coefficient using approximate formulas obtained by mathematically processing a large array of data created by systematically solving the Colebrook and White formula in the appropriate ranges of Reynolds numbers Re and relative roughness ε. The author has proposed two mathematical expressions for determining the hydraulic drag coefficient, which can be used in two steps to improve the accuracy of calculations:

$$\lambda^{I} = a^{I}\lambda_{\infty}, \tag{32}$$

and

$$\frac{1}{\sqrt{\lambda^{II}}} = -2\,lg\left(\frac{2.51}{Re\sqrt{\lambda^{I}}} + \frac{\varepsilon}{3.71D}\right), \tag{33}$$

where

$$\frac{1}{\sqrt{\lambda_{\infty}}} = -2\,lg\left(\frac{\varepsilon}{3.71D}\right); \tag{34}$$

$$a^{I} = 1 + 0.006\left(\frac{\varepsilon}{D}\right)^{-0.203}\left[lg\left(\frac{0.860}{\left(\frac{\varepsilon}{D}\right)^{0.0985}}\right)\right]^{lg\left(\frac{Re}{Re''}\right)}; \tag{35}$$

$$Re'' \approx \frac{70\sqrt{8}}{\sqrt{\lambda_{\infty}}\frac{\varepsilon}{D}}. \tag{36}$$

According to the research of G.B. Ferreri (2024), the use of only formula (32) in combination with (34-36) to calculate the hydraulic resistance coefficient allows for 93% of cases to obtain an accuracy of calculations with an error of up to 3%, the use of formula (33) additionally increases the accuracy of the results obtained with an error of up to 0.79%. P. Praks & D. Brkić (2020) proposed two explicit approximations of the Colebrook and White equation, obtained using the Lambert W-function and the Wright Omega function, with a relatively small error and short machine calculation time. In the work of D. Brkić (2024), taking into account the research presented in P.R. De Souza Mendes (2024), new approaches to solving practical problems of determining pressure losses $\rho g\Delta h$ in hydraulic systems using the classical Moody diagram, rebuilt in the coordinates of Reynolds numbers and a modified hydraulic resistance coefficient are considered:

$$\lambda^* = \frac{\rho g \Delta h D^2}{32\mu V L}, \tag{37}$$

which for the laminar regime of fluid motion is taken equal to 1, and for the turbulent regime, according to the Colebrook and White formula, is determined by:

$$\frac{\sqrt{Re}}{\sqrt{\lambda^*}} = -16lg\left(\frac{0.314}{Re}\frac{\sqrt{Re}}{\sqrt{\lambda^*}} + \frac{\varepsilon/D}{3.71}\right), \tag{38}$$

where μ, V, L – are, respectively, the dynamic viscosity coefficient, fluid velocity, and length of the hydraulic system. M. Khlapuk *et al.* (2021) used the dimensional analysis method to process the results of Nikurajze's experimental studies and proposed the following formula for the hydraulic drag coefficient in the zone of hydraulically smooth pipes in the turbulent regime:

$$\lambda = 64\left(\frac{0.01034}{Re^{0.5}} + \frac{0.03124}{Re^{0.5}} + 0.0000726\right). \tag{39}$$

According to M. Khlapuk *et al*. (2021), the results of the calculation using this formula are in better agreement with Nikuradse's experimental data. However, the formula works only in one friction zone of the turbulent regime. In M.M. Khlapuk *et al.* (2019), the researchers developed the theoretical foundations of fluid movement in a pipeline, proposing a solution to the following equation for the distribution of the average fluid velocity in a turbulent regime:

$$u_{xt} = \lambda\, Re^2 \frac{\nu^2}{64\nu_{tot} r_o^3}(r_o^2 - r^2), \tag{40}$$

where ν is the molecular kinematic viscosity of the fluid; ν_{tot} is the total kinematic viscosity, which takes into account the molecular kinematic viscosity on the pipeline wall and the kinematic turbulent viscosity between the flow layers; r_o is the pipe radius; r is the arbitrary radius of the fluid flow layer. The authors of the paper proposed to use the Hofer's formula when performing thermal and hydraulic calculations of main pipelines for the transportation of liquid hydrocarbons, which, unlike the explicit approximations of the Colebrook and White formula recommended by other authors above, allows determining the hydraulic resistance coefficient using one dependence for the entire range of turbulent fluid flow, while also ensuring accurate matching of numerical values of this variable at the transition between laminar and turbulent modes. The application of the Hofer's formula made it possible to develop a methodology for calculating the thermal and hydraulic parameters of main oil and oil product pipelines, taking into account the factors of non-isothermal flow, which in most cases are not taken into account when transporting liquids characterised by Newtonian properties. The method proposed by the authors was tested on the basis of multivariate calculations and analysis of the results obtained.

Conclusions

A method and software for the thermal and hydraulic calculation of a pipeline with regard to the non-isothermal mode of oil and oil products transportation has been developed. The method is applicable to any product flow regimes in the pipeline, including laminar, transient and all three turbulent zones. It involves the application of the generalised Hofer's formula to determine the hydraulic drag coefficient for all friction zones of the turbulent regime and takes into account the impact of flow friction heat on the energy consumption of product transportation.

The introduction of variable mode coefficients in the generalised Leibenzon's model for the hydraulic resistance coefficient makes it possible to unify the method of thermal and hydraulic calculation of pipelines in a wide range of Reynolds numbers and values of pipe surface roughness. It has been found that in the case of applying the Hofer's formula to calculate the hydraulic resistance coefficient in pipelines of any diameter, the dependence of the mode coefficients in the Leibenzon's mathematical model on the Reynolds number and relative roughness with an approximation reliability of more than 98% can be described by polynomials of the second and third order, as well as by the power function.

It has been established that taking into account the non-isothermicity caused by the difference between the initial oil temperature and the ground temperature, and the generation of frictional heat in the flow allows to specify hydrodynamic pressure losses by 14-19% depending on the degree of pipeline loading. The proposed methodology and the established models for the mode coefficients can be used in further studies related to the operation of pipelines of complex configuration, which implement various technologies and modes of pumping liquid hydrocarbons characterised by both Newtonian and anomalous rheological properties.

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Conflict of Interest

None.

References

[1] Allen, C., Holcomb, C., Zamotorin, R., & Kurz, R. (2021). Optimal parameter estimation for efficient transient pipeline simulation. In *PSIG annual meeting* (article number 2118). Virtual: PSIG.

[2] Bekibayev, T., Zhapbasbayev, U., Ramazanova, G., & Bossinov, D. (2021). Oil pipeline hydraulic resistance coefficient identification. *Cogent Engineering*, 8(1). doi: 10.1080/23311916.2021.1950303.

[3] Benavides-Muñoz, H.M. (2024). Modification and improvement of the churchill equation for friction factor calculation in pipes. *Water*, 16(16), article number 2328. doi: 10.3390/w16162328.

[4] Benner, P., Grundel, S., Himpe, C., Huck, C., Streubel, T., & Tischendorf, C. (2018). Gas network benchmark models. In S. Campbell, A. Ilchmann, V. Mehrmann & T. Reis (Eds.), *Applications of differential-algebraic equations: Examples and benchmarks. Differential-algebraic equations forum* (pp. 171-197). Cham: Springer. doi: 10.1007/11221_2018_5.

[5] Brkić, D. (2024). Revised friction groups for evaluating hydraulic parameters: Pressure drop, flow, and diameter estimation. *Journal of Marine Science and Engineering*, 12(9), article number 1663. doi: 10.3390/jmse12091663.

[6] Daneshvar, F.A., Talebbeydokhti, N., Dehghan, S.M., & Elhamian, S.M.M. (2023). Evaluation of Darcy-Weisbach friction factors of fiberglass pipes based on internal surface roughness measurement. *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, 47(3), 1749-1762. doi: 10.1007/s40996-023-01071-4.

[7] De Souza Mendes, P.R. (2024). A note on the moody diagram. *Fluids*, 9(4), article number 98. doi: 10.3390/fluids9040098.

[8] DNTD 2-86. (1986). *Standards of technological design of main oil pipelines*. Retrieved from https://online.budstandart.com/ua/catalog/doc-page?id_doc=6175.

[9] Fang, X., Xu, L., Chen, Y., & Chen, W. (2020). Correlations for friction factor of turbulent pipe flow under supercritical pressure: Review and a new correlation. *Progress in Nuclear Energy*, 118, article number 103085. doi: 10.1016/j.pnucene.2019.103085.

[10] Ferreri, G.B. (2024). A new approach for explicit approximation of the Colebrook-White formula for pipe flows. *Journal of Hydroinformatics*, 26(7), 1558-1571. doi: 10.2166/hydro.2024.280.

[11] Fiorillo, F., Esposito, L., Ginolfi, M., & Leone, G. (2024). New insights into turbulent and laminar flow relationships using Darcy-Weisbach and Poiseuille laws. *Water*, 16(10), article number 1452. doi: 10.3390/w16101452.

[12] Gallardo, A.O., Rojas, R.G., & Guerra, M.A. (2021). New explicit correlation to compute the friction factor under turbulent flow in pipes. *Brazilian Journal of Agricultural and Environmental Engineering*, 25(7), 439-445. doi: 10.1590/1807-1929/agriambi.v25n7p439-445.

[13] Hafsi, Z. (2021). Accurate explicit analytical solution for Colebrook-White equation. *Mechanics Research Communications*, 111, article number 103646. doi: 10.1016/j.mechrescom.2020.103646.

[14] Kaseng, F.L., Bayona, R., & Rodriguez, C. (2020). Alternate formula for calculating the Darcy coefficient in turbulent flow in pipes. *3CTecnología*, 9(3), 99-109. doi: 10.17993/3ctecno/2020.v9n3e35.99-109.

[15] Khlapuk, M., Bezusyak, O., Volk, L., & Zhang, Z. (2021). Theoretical research of friction factor in hydraulically smooth pipes. *E3S Web of Conferences*, 280, article number 10009. doi: 10.1051/e3sconf/202128010009.

[16] Khlapuk, M.M., Moshinskyi, V.S., Bezusiak, O.V., & Volk, L.R. (2019). To development of the water turbulent flow theory in pipes. *Bulletin National University of Water and Environmental Engineering*, 3(87), 3-16.

[17] Li, H., Fan, T., Wang, K., Long, X., He, Y., Wang, M., Cheng, W., Huang, Q., Huang, H., & Yu, W. (2024). An optimal flow rate allocation model of the oilfield treated oil pipeline network. *Petroleum*, 10(1), 93-100. doi: 10.1016/j.petlm.2023.11.001.

[18] Liu, E., Lai, P., Peng, Y., & Chen, Q. (2022). Research on optimization operation technology of QT oil pipeline based on the heuristic algorithm. *Energy Reports*, 8, 10134-10143. doi: 10.1016/j.egyr.2022.08.010.

[19] López-Silva, M., Carmenates-Hernández, D., Delgado-Hernández, N., & Chunga-Bereche, N. (2024). Explicit pipe friction factor equations: Evaluation, classification, and proposal. *Revista Facultad de Ingeniería*, 111, 38-47. doi: 10.17533/udea.redin.20230928.

[20] Luta, N., & Antonyuk, N. (2024). Modelling the coefficient of hydraulic resistance in pipelines for CO2 transportation. In *IV international scientific and practical conference "Theoretical and practical aspects of modern scientific research"* (pp. 134-136). Seul: CaseCo. Ltd. doi: 10.36074/logos-21.06.2024.026.

[21] Minhoni, R.T.D.A., Pereira, F.F.S., Silva, T.B.G., Castro, E.R., & Saad, J.C.C. (2020). The performance of explicit formulas for determining the Darcy-Weisbach friction factor. *Engenharia Agricola*, 40(2), 258-265. doi: 10.1590/1809-4430-Eng.Agric.v40n2p258-265/2020.

[22] Muzzo, L.E., Matoba, G.K., & Ribeiro, L.F. (2021). Uncertainty of pipe flow friction factor equations. *Mechanics Research Communications*, 116, article number 103764. doi: 10.1016/j.mechrescom.2021.103764.

[23] Praks, P., & Brkić, D. (2020). Review of new flow friction equations: Constructing Colebrook's explicit correlations accurately. *International Journal of Numerical Methods for Calculation and Design in Engineering*, 36(3), article number 41. doi: 10.23967/j.rimni.2020.09.001.

[24] Safta, C.-A., Georgescu, S.-C., & Dumitrescu, L. (2021). New approach of pressure losses in hydraulic power systems. In *2021 10th international conference on energy and environment (CIEM)* (pp. 1-5). Bucharest: IEEE. doi: 10.1109/CIEM52821.2021.9614769.

[25] Santos-Ruiz, I., López-Estrada, F., Puig, V., Torres, L., Valencia-Palomo, G., & Gómez-Peñate, S. (2021). Optimal estimation of the roughness coefficient and friction factor of a pipeline. *Journal of Fluids Engineering*, 143(5), article number 051304. doi: 10.1115/1.4049674.

[26] Serediuk, M.D., & Motruk, N.V. (2024). Estimated gas consumption in gas distribution networks. *Oil and Gas Power Engineering*, 1(41), 52-61. doi: 10.31471/1993-9868-2024-1(41)-52-61.

[27] Serediuk, M.D., & Pylypiv, L.D. (2013). *Pipeline transport of highly viscous hydrocarbons*. Ivano-Frankivsk: Ivano-Frankivsk National Technical University of Oil and Gas.

[28] Tolentino, S.L., & Campos, O.G. (2023). Correlation for the calculation of turbulent friction in pipes. *Ingenius*, 30, 54-63. doi: 10.17163/ings.n30.2023.05.

[29] Zhapbasbayev, U.K., Ramazanova, G.I., Bossinov, D.Zh., & Kenzhaliyev, B.K. (2021). Flow and heat exchange calculation of waxy oil in the industrial pipeline. *Case Studies in Thermal Engineering*, 26, article number 101007. doi: 10.1016/j.csite.2021.101007.

Застосування формули Хофера при теплогідравлічних розрахунках нафтотранспортних систем

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Анотація. Під час проєктування нових і модернізації та експлуатації існуючих нафтотранспортних систем обов'язковою процедурою є гідродинамічні розрахунки, які виконують зі застосуванням комп'ютерних методів. Реалізація нових технологій розрахунку створює умови для використання більш точних математичних моделей для опису гідродинамічних процесів, що супроводжують рух рідких вуглеводнів у трубопроводі. З цією метою, на основі теоретичних досліджень та математичного моделювання, у статті запропоновано удосконалений метод теплогідравлічного розрахунку трубопроводу для перекачування нафти та нафтопродуктів, який враховує неізотермічність режиму, спричинену відмінністю температури продукту на початку трубопроводу від температури ґрунту та виділенням тепла тертя потоку. Метод базується на застосуванні універсальної формули Хофера для визначення коефіцієнта гідравлічного опору в умовах трьох зон тертя турбулентного режиму, передбачає знаходження змінних коефіцієнтів режиму в математичній моделі Лейбензона в кожному перерізі трубопроводу, застосовує інтегральний спосіб обчислення теплових та гідравлічних втрат енергії під час транспортування нафти і нафтопродуктів. Метод придатний для створення обчислювальних алгоритмів та комп'ютерних програм проєктних та експлуатаційних розрахунків транспортування нафти і нафтопродуктів трубопроводами без підігріву продуктів та у разі застосування спеціальних технологій, що передбачають їх попередній підігрів. Виконано апробацію методу шляхом проведення багатоваріантних розрахунків та аналізу отриманих результатів. Одержано аналітичні залежності величини коефіцієнтів режиму моделі Лейбензона від числа Рейнольдса та відносної шорсткості поверхні труби, які можна застосовувати в теплогідравлічних розрахунках трубопроводів усіх стандартних діаметрів за різних значень шорсткості поверхні

Ключові слова: нафтопровід; нафтопродуктопровід; коефіцієнт гідравлічного опору; формула Лейбензона; втрати тиску від тертя; неізотермічність потоку; тепло тертя потоку